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Design and Aplications of State-Space Digital Filters

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Abstract

A very simple algorithm for the design of a second-order state-space digital filter has been developed. The design equations presented here were derived on the condition that the sensitivity of the coefficients in the state-space digital filter (SDF) are spread uniformly to all coefficients of the SDF. The design algorithm has been shown to provide SDF realizations having a minimum output roundoff noise while preserving low coefficient sensitivity.

Keywords: Filter Design, Filter Structures, State-Space Filters

Resumen

Se ha desarrollado un algoritmo muy simple para el diseño de filtros digitales de estado-espacio de segundo orden. Las ecuaciones de diseño que se presentan a continuación, se derivan bajo la condición de que la sensibilidad de los coeficientes en éstos filtros digitales (SDF) se distribuyen uniformemente en todos los coeficientes de los filtros, el algoritmo de diseño se muestra para dar a las actividades de SDF una mínima salida de ruido aleatorio, preservando la sensibilidad de los bajos coeficientes.

Descriptores: diseño de filtros, estructuras de filtros, filtros de estado-espacio.

Introduction

State-space structures of digital filters play an important role in the digital filter theory. Knowledge of filter coefficients yields an immediate direct form realization. However, such a realization can produce inaccuracy which is greater than in other realizations. SDFs have more complicated structre and more coefficients in comparison to direct form structures, but the main advantage is a lower roudoff noise sensitivity.

In this paper is presented an original algorithm of SDF design (Pšenička *et al.*, 1998), (Pšenička *et al.*, 1991) and (Pšenička and Zadák, 1991) based on a sensitivity analysis of zeros and poles of the second-order transfer function of the SDF.

The synthesis problem for fixed-point digital filters is more than just the specification of a transfer function or some equivalent description. The synthesis problem is to determine filter structure which minimize effects due to finite word length arithmetic. The synthesis is trivial by the absence of finite word length effects.

Finite word length effects may be devided into two categories. Coeffecient quantization has the effect altering the frequency response (Jackson, 1970), (Liu, 1971). This effect will not be considered here. The second category are effects due to quantization of arithmetic opperations, roundoff of the results of a multiplication (Bose and Brown, 1990), (Liu, 1971) (Mills *et al.*, 1981) overflow of internal storage registers by summation (Lecler and Bauer, 1992), (Jackson, 1970), (Liu, 1971).

The impulse response of a stable time-invariant linear recursive digital filter asymptotically approaches the zero value. However, accumulator overflow and multiplication rounding introduce nonlinearities that result in creating limit cycles. Overflow limit cycles have generally large amplitude but, fortunately, it is possible to eliminate them. Limit cycles resulting from nonlinearities introduced due to multiplication rounding have relatively smaller values, but they cannot be eliminated so easily. This limit cycles have been studied for digital filters using fixed-point arithmetic

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(Jackson and Hill, 1970), (Bose and Brown, 1990), (Mullis and Roberts, 1976).

Synthesis of Digital State-Space Filter

The N-th order single input/output Digital State Space Filter (SDF) can be described by o equations

$$\mathbf{u}\,(n+1)=\mathbf{A}\mathbf{u}\,(n)+\mathbf{B}x\,(n)$$

$$y\,(n)=\mathbf{C}\mathbf{u}\,(n)+\mathbf{D}x\,(n) \tag{1}$$

where **u** (n) is the N dimensional vector of state variables, x(n) is an input sequence, y(n) is an output sequence and the state-matrixes **A**, **B**, **C** y **D** contain the filter coefficients. The matrix **A** represents a system matrix of the dimension Nx N. The associated system function H(z) is given by

$$\mathrm{H}(z)=\mathbf{D}+\mathbf{C}(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \tag{2}$$

where **I** is the identity matrix.

Second Order State-Space Structure

The recursive second-order SDF can be described by the state matrices **A**,**B**, **C** and **D**

$$\mathbf{A}=\begin{bmatrix} a_{11}a_{12} \\ a_{21}a_{22} \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\mathbf{C}=[c_1\ c_2] \qquad \mathbf{D}=[\mathrm{d}] \tag{3}$$

The equation (1) for the second-order state filter can be writen in the form

$$\begin{bmatrix} u(n+1) \\ u(n+2) \end{bmatrix}=\begin{bmatrix} a_{11}a_{12} \\ a_{21}a_{22} \end{bmatrix}\begin{bmatrix} u_1(n) \\ u_2(n) \end{bmatrix}+\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}x(n)$$

$$y(n)=[c_1c_2]\begin{bmatrix} u_1(n) \\ u_2(n) \end{bmatrix}+dx(n) \tag{4}$$

The second-order SDF structure derived from the state-equations (4) is shown in Fig. 1a. The common system function of the second order canonic digital filter (CDF) Fig. 1b is given by

$$H(z)=\frac{B_0+B_1z^{-1}+B_2z^{-2}}{1+A_1z^{-1}+A_2z^{-2}} \tag{5}$$

or equivalently

$$H(z)=B_0+\frac{(B_1-B_0A_1)z^{-1}+(B_2-B_0A_2)z^{-2}}{1+A_1z^{-1}+A_2z^{-2}} \tag{6}$$

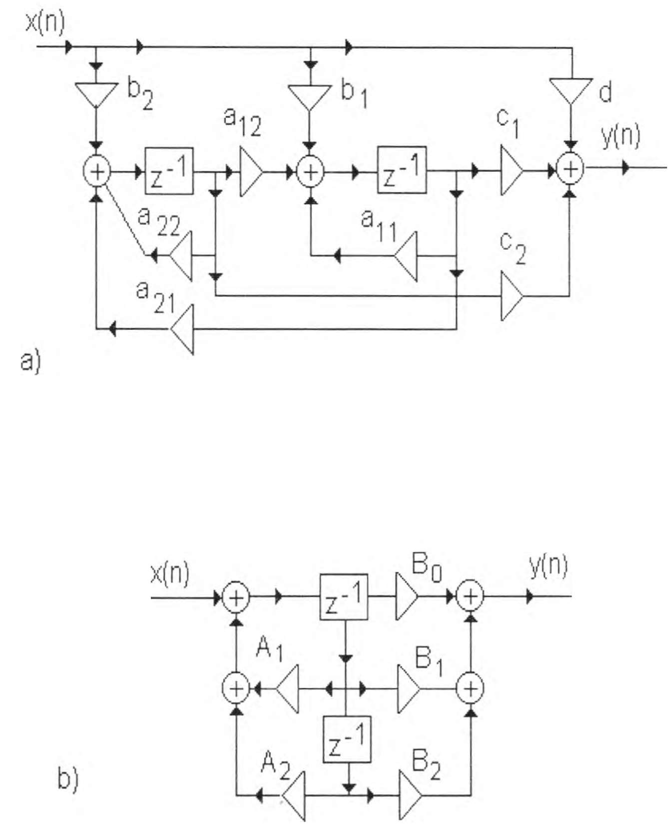

Figure 1. State-space filter and second-order direct canonical structure

Substituting matrices (3) in the equation (2) can imply the system function H(z) of SDF in the form

$$H(z)=d+\frac{\alpha_1z^{-1}+\alpha_2z^{-2}}{1+\beta_1z^{-1}+\beta_2z^{-2}} \tag{7}$$

where constantsand are expressed as:

$$\alpha_1=b_1c_1+b_2c_2$$
$$\alpha_2=b_1c_2a_{21}+b_2c_1a_{12}-b_1c_1a_{22}-b_2c_2a_{11}$$
$$\beta_1=-(a_{11}+a_{22})=-tr\mathbf{A}$$
$$\beta_2=(a_{11}a_{22}-a_{12}a_{21})=det\ \mathbf{A}$$

Symbols tr**A** and det**A** denote the trace and the determinant of a system matrix **A**, respectively. Comparing H(z) of the equations (6) y (7) we get five equations for the computation of nine state-space filter coefficients. The next necessary equations follow, for example from the relations for sensitivities of zeros and poles of transfer

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functions to the filter coefficients, as it was derived in (Bomar and Joseph, 1987) and (Jackson and Hill, 1970). As equations (6) and (7) are expected to be equal the following equations have to hold for the system function of the second order

$$B_0 = d$$

$$B_1 = b_1 c_1 + b_2 c_2 - det\mathbf{A}$$

$$B_2 = d.det\mathbf{A} + b_1 c_2 a_{21} + b_2 c_1 a_{21} - b_1 c_1 a_{22} - b_2 c_2 a_{11}$$

$$A_1 = -tr\mathbf{A}$$

$$A_2 = det\mathbf{A}$$

The relations between coefficients of state-space filters (7) and coefficients of direct canonical structure (6) for $a_{11} = a_{22}$, $a_{12} = -a_{21}$, $c_2 = -b_2$ and $b_1 = c_1$ are given in (Pšenička *et al.*, 1998) by the following equations:

$$\begin{aligned}
d &= B_0 \\
a_{11} &= a_{22} = -A_1/2 \\
a_{12} &= -a_{21} = \sqrt{A_2 - \frac{A_1^2}{4}} \\
c_2 &= -b_2 = \sqrt{\frac{\sqrt{\alpha^2 - 4e - \alpha}}{2}} \\
c_1 &= b_1 = -\sqrt{\alpha + c^2} \\
\alpha &= B_1 - B_0 A_1 \\
\beta &= B_2 - B_0 A_2 \\
e &= \frac{(\beta + \alpha a_{22})^2}{4 a_{12}^2}
\end{aligned} \tag{10}$$

Another relations between coefficients of SDF and CDF for $a_{11} = a_{22}$, $a_{12} = -a_{21}$, $b_1 = c_2$ and $b_2 = c_1$ takes the form:

$$\begin{aligned}
d &= B_0 \\
a_{11} &= a_{22} = -A_1/2 \\
a_{12} &= -a_{21} = \sqrt{A_2 - A_1^2/4} \\
\alpha &= B_1 - B_0 A_1 \\
\beta &= B_2 - B_0 A_2 \\
\gamma &= (a_{11}\alpha + \beta + \sqrt{(a_{11}\alpha + \beta)^2 + a_{12}^2 \alpha^2})/2a_{12} \\
c_1 &= b_2 = \sqrt{\gamma} \\
c_2 &= b_1 = \alpha/2c_1
\end{aligned} \tag{11}$$

The set of equations (10) and (11) represent algorithms for the synthesis of unscaled second order real coefficient SDFs. When required, a scaled realization can always be obtained from an unscaled one by applying the transformations (Bomar, 1989).

$$\begin{aligned}
\mathbf{A'} &= \mathbf{T}^{-1}\mathbf{AT} \\
\mathbf{B'} &= \mathbf{T}^{-1}\mathbf{B} \\
\mathbf{C'} &= \mathbf{T}^{T}\mathbf{C} \\
\mathbf{D'} &= \mathbf{D}
\end{aligned} \tag{12}$$

where **A'**, **B'**, **C'** and **D'** are the state matrices of the new (scaled) realization, and **T** is a nonsingular diagonal (scaling transformation) matrix of the form

$$\mathbf{T} = \begin{bmatrix} t_{11} & 0 \\ 0 & t_{22} \end{bmatrix} \tag{13}$$

The diagonal elements t_{11} and t_{22} are given by the *Lp* norms of the transfer functions (14) and (15), respectively, where the transfer function $F_i(z)$ from the *ith* state variable node takes the value of (14)

$$F_1(z) = \frac{b_1 z^{-1} + (a_{12} b_2 - a_{22} b_1) z^{-2}}{1 - tr\mathbf{A} z^{-1} + det\mathbf{A} z^{-2}} \tag{14}$$

and (15)

$$F_2(z) = \frac{b_2 z^{-1} + (a_{21} b_1 - a_{11} b_2) z^{-2}}{1 - tr\mathbf{A} z^{-1} + det\mathbf{A} z^{-2}} \tag{15}$$

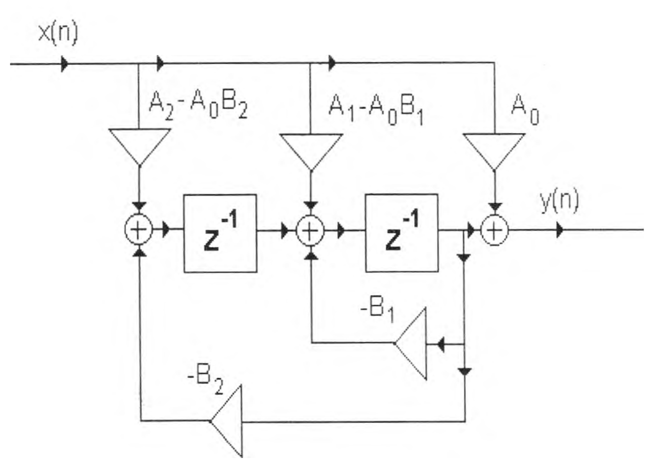

Figure 2. a) Filter of the second order (direct realization from II)

for the first and second state variable node, respectively. In fact, the value of $p = 2$ is usually applied to the design

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of broad-band DF's, while the value of $p=\infty$ has proved to be appropriate when narrow-band DFs are synthesized.

It can be shown that the scaling constraints (12) when appropriately handled, place two additional constraints upon the SDF coefficients, thus leaving two degrees of freedom for further (scaled) design.

For example, the constraining requirements

$$\begin{aligned} a_{22} &= 0 \\ c_2 &= 0 \end{aligned} \tag{16}$$

lead to the design of the canonical (though scaled) structure in Fig. 2. Another example, characterized by (Jackson *et al.*, 1979) equations (18)

$$\begin{aligned} a_{22} &= a_{11} \\ b_1c_1 &= b_2c_2 \end{aligned} \tag{17}$$

gives as result the design of low noise, fixed-point, second order state-space structures.

Straightforward design quations for low-roundoff-noise state-space structure have been published in (Bomar,1985). Among the algorithms (10) and (11), the design eqs. (11) deserve special attention due to the following:

Consider the design constraints (17) yielding the synthesis of minimum roundoff noise scaled SDF's. From(11), indubitable the following equations hold for this unscaled-case

$$\begin{aligned} a_{11} &= a_{22} \\ b_1c_1 &= b_2c_2 \end{aligned} \tag{18}$$

It is easy to see, from (13) and (12), that

$$\begin{aligned} a'_{11} &= a_{11} \\ a'_{22} &= a_{22} \\ b'_1 &= b_1 / t_{11} \\ b'_2 &= b_2 / t_{22} \\ c'_1 &= c_1 t_{11} \\ c'_2 &= c_2 t_{22} \end{aligned} \tag{19}$$

By the way of substituting (19) into (18)

$$\begin{aligned} a'_{11} &= a'_{22} \\ b'_1c'_1 &= b'_2c'_2 \end{aligned} \tag{20}$$

Therefore, the algorithm (11), developed on the basis of a SDF coefficient sensitivity analysis, yields (after scaling) the design equations for the synthesis of minimum roundoff noise second order scaled SDFs. However, keeping the recent aforementioned developments by Smith *et al.* (1992) in view, the scaling (12) is not essential, when floating-point arithmetic is going to be applied to the SDF implementation. Then, an unscaled realization obtained through (11) can be directly used without any degradation of the minimum output roundoff noise property.

Moreover, consider the root stability condition for the second order SDF´s

$$\left\| z_{\infty 1,2} \right\| < 1 \tag{21}$$

In (Mills *et al.* 1981) a condition for a second order SDF realization without limit cycles has been derived taking the form of the inequality

$$\left\| a_{11} - a_{22} \right\| + \left\| z_{\infty 1,2} \right\|^2 \le 1 \tag{22}$$

Because of (21) and (18) or (20), this inequality is always satisfied by the coefficients of a stable SDF structure computed using the algorithm (11).

As an example we shall realize the state space digital notch filter of sixth order having the transfer function

$$\begin{aligned} H(z) &= \frac{1-0.125581039z^{-1}+z^{-2}}{1-0.064723164z^{-1}+0.98157085z^{-2}} \\ &x\frac{1-0.125581039z^{-1}+z^{-2}}{1-0.121635794z^{-1}+0.938155107z^{-2}} \\ &x\frac{1-0.125581039z^{-1}+z^{-2}}{1-0.184000618z^{-1}+0.98157085z^{-2}} \end{aligned} \tag{23}$$

Using the set of equations (10), the values of the state space digital filters in the cascade form are

$b_{21}=-0.249$	$b_{22}=-0.184$	$b_{23}=-0.049$
$b_{11}=-0.033$	$b_{12}=-0.173$	$b_{13}=-0.247$
$d_1=1.0000$	$d_2=1.0000$	$d_3=1.0000$
$a_{111}=0.0323$	$a_{112}=0.0608$	$a_{113}=0.0920$
$a_{121}=0.9902$	$a_{122}=0.9666$	$a_{123}=0.9864$
$a_{211}=-0.990$	$a_{212}=-0.967$	$a_{213}=-0.986$
$a_{221}=0.0323$	$a_{222}=0.0608$	$a_{223}=0.0920$
$c_{21}=0.2489$	$c_{22}=0.1841$	$c_{23}=0.0489$

State digital filters in canonic form Fig. 3 have been implemented with the signal processors TMS320C25. By means of the simulator we have obtained the samples of the impulse response and from the 40 samples via FFT, we have get the result that is presented in figure 4.

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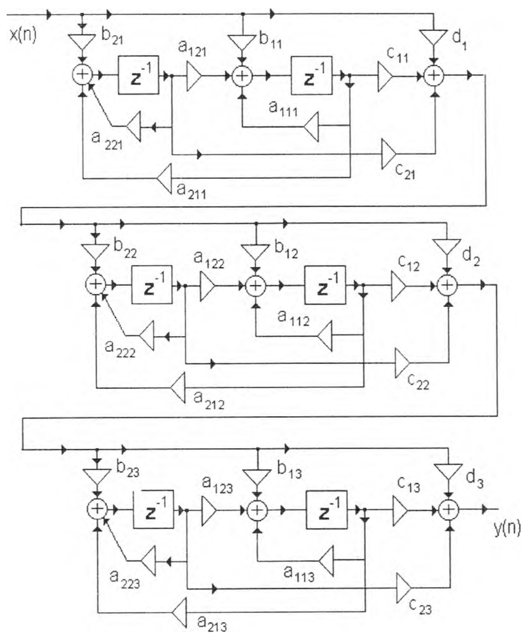

Figure 3. Six order Notch state-space filter in casacade form

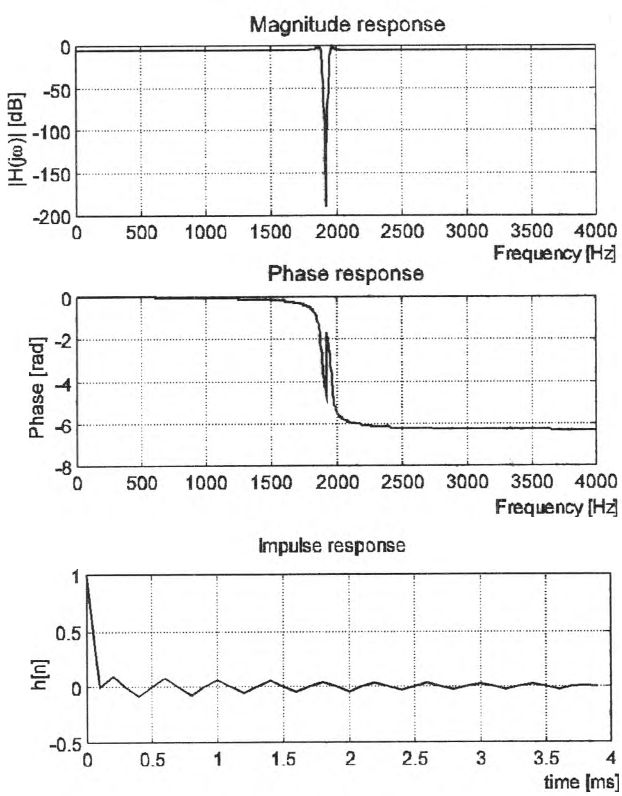

Figure 4. Frequency response of the state digital filter

Design of n-order State-Space Filter

In this section we shall derive the state-space structure of the third order filter. The state matrices of the third order state-space filter take the form

$$\mathbf{D}=d \qquad \mathbf{C}=[c_1\ c_2\ c_3]$$

$$\mathbf{B}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{24}$$

By the way of substitution of (24) in (25) we obtain the state-flow matrix (26)

$$\mathbf{N}_e=\left[\begin{array}{c|c|c} \mathbf{D} & -\mathbf{I} & \mathbf{C} \\ \hline z^{-1}\mathbf{B} & 0 & z^{-1}\mathbf{A}-\mathbf{I} \end{array}\right] \tag{25}$$

$$\mathbf{N}_e^{(5)}=$$

$$\begin{bmatrix} d & -1 & c_1 & c_2 & c_3 \\ z^{-1}b_1 & 0 & -1+a_{11}z^{-1} & a_{12}z^{-1} & a_{13}z^{-1} \\ z^{-1}b_2 & 0 & a_{21}z^{-1} & -1+a_{22}z^{-1} & a_{23}z^{-1} \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & -1+a_{33}z^{-1} \end{bmatrix} \tag{26}$$

To expand the state-flow matrix (26) which contains five columns and four rows in the matrix (28) with six columns and five rows, we use the equation (27).

$$n_{ij}^{n}=n_{ij}^{n-1}-n_{in}^{n}.n_{nj}^{n} \tag{27}$$

If we choose the elements of the new matrix $\mathbf{N}^{(6)}$, $n_{26}^{(6)}=n_{46}^{(6)}=n_{56}^{(6)}=0$, then the first, third and fourth rows in the new matrix $\mathbf{N}^{(6)}$ remain unchanged.

$$\mathbf{N}^{(6)}=$$

$$\begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 \\ n_{31}^{(6)} & n_{32}^{(6)} & n_{33}^{(6)} & n_{34}^{(6)} & n_{35}^{(6)} & n_{36}^{(6)} \\ z^{-1}b_2 & 0 & a_{21}z^{-1} & -1+a_{22}z^{-1} & a_{23}z^{-1} & 0 \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & 1-a_{33}z^{-1} & 0 \\ n_{61}^{(6)} & n_{62}^{(6)} & n_{63}^{(6)} & n_{64}^{(6)} & n_{65}^{(6)} & n_{66}^{(6)} \end{bmatrix} \tag{28}$$

The elements of the matrix (28), $n_{61}^{(6)}, n_{62}^{(6)}, n_{63}^{(6)}, n_{64}^{(6)}, n_{65}^{(6)}, n_{66}^{(6)}$ and $n_{36}^{(6)}$ can be chosen and the remaining elements $n_{31}^{(6)}, n_{32}^{(6)}, n_{33}^{(6)}, n_{34}^{(6)}$ and $n_{35}^{(6)}$ can be obtained by means of the equation (27). If we choose

$$n_{26}^{(6)}=0 \quad n_{46}^{(6)}=0 \quad n_{56}^{(6)}=0 \quad n_{62}^{(6)}=0$$

$$n_{66}^{(6)}=-1 \quad n_{65}^{(6)}=a_{13} \quad n_{64}^{(6)}=a_{12} \quad n_{63}^{(6)}=a_{11}$$

$$n_{36}^{(6)}=z^{-1} \quad n_{61}^{(6)}=b_1$$

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then the elements of the new matrix take the form

$$\begin{aligned}
n_{31}^{(6)} &= n_{31}^{(5)} - n_{36}^{(6)} n_{61}^{(6)} = z^{-1} b_1 - z^{-1} b_1 &&= 0 \\
n_{32}^{(6)} &= n_{32}^{(5)} - n_{36}^{(6)} n_{62}^{(6)} = 0 - z^{-1} 0 &&= 0 \\
n_{33}^{(6)} &= n_{33}^{(5)} - n_{36}^{(6)} n_{63}^{(6)} = -1 + a_{11} z^{-1} - a_{11} z^{-1} &&= -1 \\
n_{34}^{(6)} &= n_{34}^{(5)} - n_{36}^{(6)} n_{64}^{(6)} = z^{-1} a_{12} - z^{-1} a_{12} &&= 0 \\
n_{35}^{(6)} &= n_{35}^{(5)} - n_{36}^{(6)} n_{65}^{(6)} = z^{-1} a_{13} - z^{-1} a_{13} &&= 0
\end{aligned}$$

and we can write now the matrix $\mathbf{N}^{(6)}$

$\mathbf{N}^{(6)} =$

$$\begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} \\ z^{-1} b_2 & 0 & a_{21} z^{-1} & -1 + a_{22} z^{-1} & a_{23} z^{-1} & 0 \\ z^{-1} b_3 & 0 & a_{31} z^{-1} & a_{32} z^{-1} & -1 + a_{33} z^{-1} & 0 \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & -1 \end{bmatrix} \tag{29}$$

Similarly, we can obtain the matrix $\mathbf{N}^{(7)}$ and $\mathbf{N}^{(8)}$. After a very simple calculation, we can get the signal flow matrix in the form.

$$\mathbf{N}^{(8)} = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & z^{-1} \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & -1 & 0 & 0 \\ b_2 & 0 & a_{21} & a_{22} & a_{23} & 0 & -1 & 0 \\ b_3 & 0 & a_{31} & a_{32} & a_{33} & 0 & 0 & -1 \end{bmatrix} \tag{30}$$

The digital filter structure that corresponds to signal flow matrix $\mathbf{N}^{(8)}$ is presented in the Fig.5.

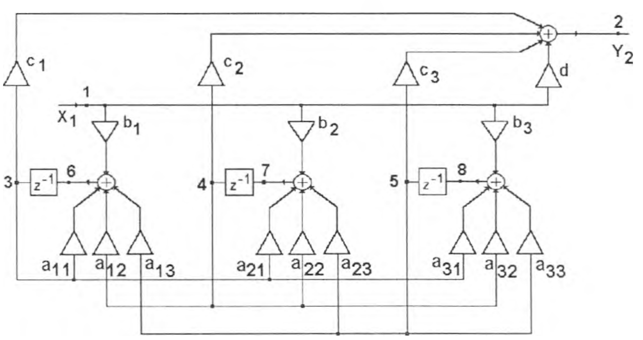

Fig. 5. Third order state-space filter

Second canonic form of the state-space digital filter can be obtained from the structure in the Fig.5, changing the sumators to nodes, the nodes to sumators, the input to output and the directions of the multipliers.

Implementation of the State-Space Filter with TMS320C30

By means of the MATLAB we can obtain the state-space matrices for elliptic aproximation if

$N=2$, $a_{max}=0.05$dB $a_{min}=20$dB
$f_1=0.4$, $f_2=0.6$

in the form

$\mathbf{C}=[0.1620\ \ 0.5713\ \ 0.1620\ \ 0.5713\ ]$
$\mathbf{D}=0.2489$
$\mathbf{B}^T= [0.3855\ \ 0.2716\ \ -0.3855\ \ -0.2716]$

$$\mathbf{A} = \begin{bmatrix} -0.5855 & -0.2955 & 0.4195 & -0.2955 \\ 0.2955 & -0.2082 & 0.2955 & 0.7918 \\ -0.4195 & 0.2955 & 0.5805 & 0.2955 \\ -0.2955 & -0.7918 & -0.2955 & 0.2082 \end{bmatrix}$$

The following equations that realize bandpass state-space filter of 4th order was programed for the DSP TMS320C30. The structure of the state-space-filter of the 4th order is presented in the figure 6.

N1=B1.XN+N2.A11+N4.A12+N6.A13+N8.A14
N3=B2.XN+N2.A21+N4.A22+N6.A23+N8.A24
N5=B3.XN+N2.A31+N4.A32+N6.A33+N8.A34
N7=B4.XN+N2.A41+N4.A42+N6.A43+N8.A44
YN=D.XN+N2.C1+C2.N4+C3.N6+C4.N8

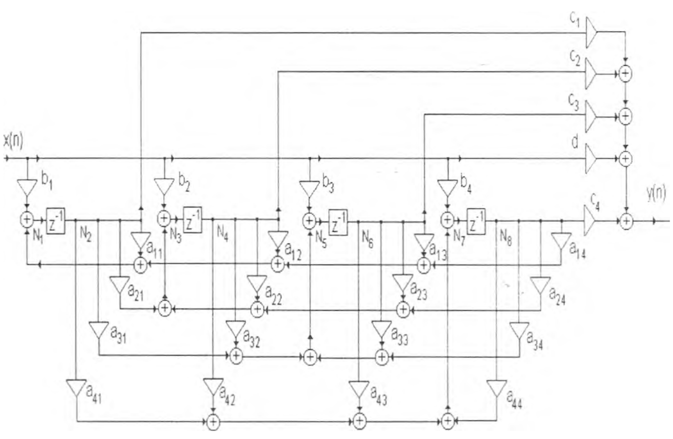

Fig. 6. State-space filter of the fourth order

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The main program for the state-space filter described in this paper, solicit the programs SSFEVMco.asm, inievm.asm and SSFEVM30.cmd. All programs that are necessary for main program can by asked by the e-mail pseboh@servidor.unam.mx.

```
;********************************
; Main program for the state space
; filter SSFEVM30
; EVALUATION MODULE TMS320C3x
;*********************************
;The scheme of filter is on Figure (1).
;The variables in program correspond
;the variables on Figure (1). The parts
;of filter (blocks) are realised by
;subroutines. The subroutines realise
;the  equation (13)
;*************************************
;
;assembler:
;ASM30 SSFEVM30.asm
;ASM30 SSFEVMco.asm
;linker:
;LNK30 SSFEVM30.cmd
;real-time processing on EVM:
;EVM30 SSFEVM30.out
;
;*************************************
; N = 2 Amax = 0.0500 Amin = 20
; f1 = 0.4000 f2 = 0.6000
;wn = 0.4000   0.6000
*************************************
;Stave Space Filter - band pass (SSF)
;
; N = 2 Amax = 0.05 Amin = 20 f1 = 0.4
; f2 = 0.6 wn = 0.4   0.6
;*************************************
;   MATLAB
;   wn=[f1 f2]
;   [a,b,c,d]=ellip(N,Amax,Amin,wn)
;*************************************

      .global _main,inievm,serialp,
      .global A11_addr,B1_addr,C1_addr,D
;
order_1    .set   3
order   .float  4
;*****************************
; the file containing filter
; coefficients file SSFcoef.asm
;*****************************
;Matrix N
N1   .float   0
N3   .float   0
N5   .float   0
N7   .float   0
N9   .float   0
N11  .float   0
N13  .float   0
N15  .float   0
N17  .float   0

N1_addr    .word   N1
N2      .float   0
N4      .float   0
N6      .float   0
N8      .float   0
N10     .float   0
N12     .float   0
N14     .float   0
N16     .float   0
N18     .float   0

N2_addr    .word   N2

serialp .word   808040h

_main   LDI     @serialp,AR3
        CALL    inievm
        AND     0h,ST
        LDF     0,R6
        LDI     010H,IE
        FIX     R6,R5
Loop: LSH     2,R5
        IDLE
        AND     0h,ST
        FLOAT   R5,R0
        LDI     @A11_addr,AR1
        LDI     @B1_addr,AR5
        LDI     @N1_addr,AR6
        LDI     @N2_addr,AR2
        LDI     @C1_addr,AR7
        LDF     @order,R1
        LDF     0,R6
Equat:
        LDI     order_1,RC
        RPTB    LoopNA
        MPYF3   *AR1++,*AR2++,R2
        ADDI    1,R3
LoopNA    ADDF    R2,R6
        MPYF3   *AR5++,R0,R4
        ADDF    R4,R6
        STF     R6,*AR6++
        LDF     0,R6
```

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```
        SUBF    1,R1
        LDI     @N2_addr,AR2
        BNZ     Equat
;YN
        LDF     @D,R7
        MPYF3   R7,R0,R6
        LDI     order_1,RC
        RPTB    LoopNC
        MPYF3   *AR7++,*AR2++,R2
LoopNC   ADDF    R2,R6

;Actualization of N even
;
        LDI     order_1,RC
        RPTB    Actual
        LDF     *—AR6,R7
Actual  STF     R7,*—AR2
        BRD     Loop
        FIX     R6,R5
        NOP
        NOP
```

The values of the impulse response obtained by the spectrum analyzer are shown in figure 7 and are identical with the impulse response obtained by the simulator and by the MATLAB (Fig. 8).

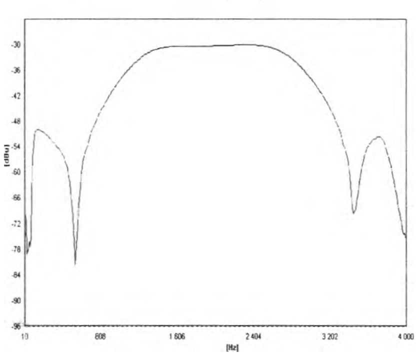

Fig. 7. Impulse response obtained by spectrum analyzer

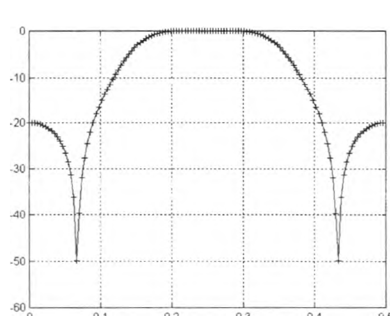

Fig. 8. Impulse response obtained by the simulator and the MATLAB

Conclusion

The state-space DF's are special structures of digital filters with lower sensitivity to roundoff effects by fixed-point implementation in comparison to canonical direct form II. The SDF's have a lower sensitivity to coefficient quantization in comparison to CDF's. The probability of occurrence of zero-input limit cycles by rounding is lower as in the case of direct form. Unfortunately, if the limit cycle occurs its amplitude can be higher (two times maximally), compared to direct form realization. Digital filters with minimum norm have asymptotically stable realization for overflow oscillations and for magnitude truncation limit cycles. The disadvantage of SDF's is a higher number of multipliers, 9 coefficients are necessary against 5 coefficients of DSF structure.

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