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Modeling of a 1kW free piston Stirling engine: Opportunity for sustainable electricity production

Modelado de un motor Stirling de pistón libre de 1kW: Oportunidad para la producción de electricidad sostenible

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Abstract

As part of the many alternatives for the development of new methods for sustainable energy transformation, the Stirling engine is distinguished due to the characteristics of an external combustion engine, with many advantages. These engines can be activated with heat, representing an additional option for the use of renewable energy. The Free Piston Stirling Engine (FPSE) is special due to the elimination of all wearing mechanisms of a typical kinematic Stirling engine. A linear mechanical storage device replaces the crank device of the kinematic one, making possible long operating life, higher efficiency, and zero maintenance. This document presents a theoretical model of a Free Piston Stirling Engine (FPSE) based on a dynamical and thermal analysis. As the core of the study, the “first-order analysis” of the thermal machine is implemented by taking some ideal assumptions. A technique of evolutionary computation is presented as a feasible solution for finding functional correct design parameters. For the dynamical part, the mathematical model is obtained through energy balances, and then a stability study is implemented to assure the oscillations of the machine. Results include several cases, which offer parameters such as cylinder and rod area, cooler temperature, heater temperature, piston and displacer mass, piston and displacer stiffness, phase angle, frequency, and power. It is critical to consider the sensibility of the system, and therefore, it is essential to apply a stability analysis in order to fully understand the behavior of the thermal machine. Results were compared in terms of a calculated error considering first-order analysis and the stability study developed with the Schmidt analysis.
Keywords: Stirling, engine, free piston, stability, genetic algorithm, design.

Resumen

Como parte de las muchas alternativas para el desarrollo de nuevos métodos para la transformación de energía sostenible, el motor Stirling se distingue por las características de un motor de combustión externa, con muchas ventajas. Estos motores se pueden activar con calor, lo que representa una opción adicional para el uso de energía renovable. El motor de pistón libre Stirling (FPSE) es especial debido a la eliminación de todos los mecanismos de desgaste de un motor cinemático típico de Stirling. Un dispositivo de almacenamiento mecánico lineal reemplaza el dispositivo de manivela del cinemático, lo que hace posible una larga vida útil, una mayor eficiencia y un mantenimiento nulo. Este documento presenta un modelo teórico de un motor Stirling de pistón libre (MSPL) basado en un análisis dinámico y térmico. Como el núcleo del estudio, el “análisis de primer orden” de la máquina térmica se implementa tomando algunas suposiciones ideales. Una técnica de computación evolutiva se presenta como una solución factible para encontrar parámetros de diseño correctos y funcionales. Para la parte dinámica, el modelo matemático se obtiene a través de balances de energía, y luego se implementa un estudio de estabilidad para asegurar las oscilaciones de la máquina. Los resultados incluyen varios casos, que ofrecen parámetros como el área del cilindro y la varilla, la temperatura del enfriador, la temperatura del calentador, la masa del pistón y del desplazador, la rigidez del pistón y del desplazador, el ángulo de fase, la frecuencia y la potencia. Es fundamental tener en cuenta la sensibilidad del sistema y, por lo tanto, es esencial aplicar un análisis de estabilidad para comprender completamente el comportamiento de esta máquina térmica. Los resultados se comparan en términos de error calculado considerando el análisis de primer orden y el estudio de estabilidad desarrollado con el análisis de Schmidt.
Descriptores: Stirling, motor, pistón libre, estabilidad, algoritmo genético, diseño.

Introduction

The demand for energy has raised in the modern world due to overpopulation. This scenario, together with the shortage of fossil fuels, have given rise to the development of new methods for energy transformation during the past decades. The primary goal is to utilize the higher amount of energy available inside the fuel or source, which will result in technologies that are more efficient. For the past decades, the boom of renewable energy has exponentially increased with solar panels and wind turbines as the most known technologies, but there are many others that are trying to join the seek of solutions, like the wave energy converters. The topic of this document is the Stirling engine which has the characteristics of an external combustion engine but the advantage of clean energy conversion.

The Stirling engine was invented in 1816 by the Reverend Robert Stirling (Walker, 1980), at that time the machine was better known as a "Hot Air" engine. Although for that time the device occupied relatively unimportant role among other engines, now the Stirling engines rearise due to their high efficiency, their capacity to use any source of heat (including solar energy), their quiet operation, and non-polluting character. These represent a notable advantage to the energetic crisis, and that is the reason for the increasing number of works focused on developing this technology.

A Stirling engine is a thermo-mechanical device that operates on a closed regenerative thermodynamic cycle (there is no change in mass inside the engine) with cycle compression and expansion of the working fluid (Figure 1) at different temperature levels (Martini, 1983). This last description includes a wide variety of configurations, functions, and characteristics. It include both reciprocating and rotary systems generated by different simple and complex mechanisms. They are classified into kinematic (beta, alpha and gamma) and dynamic, which includes free piston Stirling engines. A kinematic Stirling engine uses a classic crank mechanism with a flywheel to produce a 90°out of phase reciprocating motion of the piston parts. These type of Stirling engines are the most studied until now and there are well-resumed works on both theoretical and experimental studies (Ipci & Karabulut, 2018; Cheng & Chen, 2017; Altin *et al.*, 2018; Sowale *et al.*, 2018; Çınar *et al.*, 2018, among others).

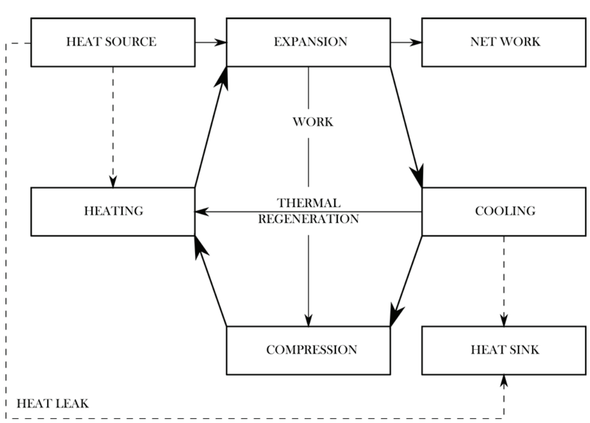

Figure 1. Process for regenerative heat engines

The micro-cogeneration (μCHP) is one of the principal applications, although there are many others like vehicle propulsion to produce a zero or low level of pollution, to directly pump blood, generate electricity, or generate hydraulic power, as summarized by Valenti *et al.* (2015). Some articles (Zhu *et al.*, 2018; Remiorz *et al.*, 2018) investigated if a μCHP, based on a Stirling engine, life-cycle costs, emissions, and energy requirements in a domestic house. Conroy *et al.* (2014) found that operating a WhisperGen Mk IV Stirling engine unit in the house resulted in an energy saving of 2063 kW h of electricity and generated a thermal output of 20095 kW h during the trial period, operated for 2512 h and produced an annual saving of 180 euros; also there was a decrease in the CO_2 emissions by 1040 kg CO_2 per annum. The interest in developing this kind of technology has been growing not only as a common subject for research but as a device in which companies can make a profit. That is the case of United Stirling of Sweden with his P-75, 75kW truck engine and Sunpower of Athens, Ohio with his atmospheric air engine which can produce 850 watts, to mention a few (Walker, 1980).

The FPSE is distinctive for the reason that it excludes all wearing mechanisms related to a kinematic Stirling engine and thus eliminates the need for lubricant. It replaces the crank mechanism of the kinematic Stirling engine by a linear mechanical storage device such as a compression spring. FPSE provides the possibility of an extremely long operating life, higher efficiency and zero maintenance (Deetlefs, 2014). Some works are found in the literature, showing the use of renewable energies using FPSE (Motamedi *et al.*, 2018; Langdon *et al.*, 2017; Bartela *et al.*, 2018). Other researchers are interested in the design, but fixing parameters like frequency (Zare *et al.*, 2016) or using scaling laws (Formosa & Fréchette, 2013).

The successful design of a Stirling engine, and particularly FPSE, requires accounting the complex interaction between different types of processes, being the principal: the heat transfer at the different heat exchangers, the mechanical forces that link the different piston-displacer movements, which determine the dynamics of the engine, and the thermodynamics of the working fluid. It is for this reason that different approaches, with different simplifications, have been developed. These can be classified into: zero-order methods, which proposed simplified and empirical correlations to estimate the power output of engines; first-order methods that assume ideal heat transfer processes at the heat exchangers; second-order methods that include the heat transfer limitations at the heat exchangers; third-order methods that assuming one dimensional variations apply balances at a number of control volumes; and multidimensional analysis which uses CFD solvers for the analysis of the engine (Dyson *et al.*, 2004).

The mentioned methods focus on the thermodynamics of the engine, and reflect good results under specified conditions, especially if the piston-displacer movements are imposed, for example by a crank mechanism. However, in the case of free piston engines, the dynamics of these components should be determined by solving additional equations that reflect these interactions. In this sense, the works of Zare *et al.* (2016), present a successful approach to integrate these phenomena, identifying that the key complication to design useful FPSE is to assure the stability of the oscillations. This is not a minor mission because of the decoupled pistons and the sensibility of some parameters. Therefore, this research aimed to continue the analysis of the coupling between the kinematic and dynamic constraints, and thermodynamics in FPSE. For this purpose, a mathematical model was developed including the dynamic equations for stability analysis of FPSE. The code allowed analyzing a set of parameters that guide the design of an engine, a power output of 1 kW was set as a design objective, and a genetic algorithm optimization was implemented to solve the optimization problem.

Development

The Stirling cycle

The Stirling cycle is an ideal thermodynamic cycle built by two isothermal curves and two isometric regenerative processes. To fully understand the complete cycle, each one of the processes is described in the following.

Let's have in mind the two pistons shown in Figure 2 on each end of the cylinder and among them a regenerator, which may be thought as a thermodynamic sponge, releasing and absorbing heat alternately.

One of the volumes between the regenerator and a piston is called the expansion space, and it's maintained at a high temperature (T_{max}). The remaining volume is named compression space kept at a low temperature (T_{min}). Therefore, the temperature gradient is $T_{max} - T_{min}$ through the longitudinal direction; moreover, it's assumed that there is no thermal conduction between these two volumes.

It is essential to get the initial configuration of the pistons and the fluid, to describe the cycle; thus, the following assumptions are made:

1. As an initial configuration, imagine the compression space piston is at the outer death point and the expansion space piston is at the inner death point (near the regenerator).
2. All the fluid is at the compression space.
3. At this position, the volume is a maximum.
4. The pressure and temperature are minimum values.

The four Stirling cycle processes are described below (Figure 2):

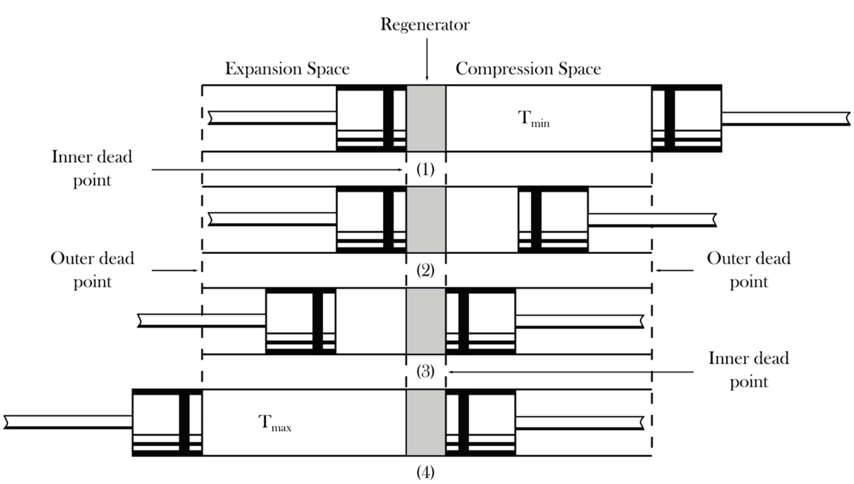

Figure 2. Stirling cycle diagram

Process 1-2: Caused by the movement of the compression piston and the lack of motion of the expansion piston, the working fluid is squeezed. At the same instant, the temperature remains constant due to the transfer of heat (Q_c) from the compression space to the surrounds, and therefore the pressure increases.

Process 2-3: Also named as transfer process, both pistons move, the compression piston towards the regenerator, and the expansion piston simultaneously away. So, the volume remains constant. Now the fluid is passing through the regenerator, the compression space to the expansion space. Also, the fluid is heated up while moving, from T_{min} to T_{max}. The gradual increase of temperature at constant volume also causes an increase in pressure.

Process 3-4: Now the expansion piston keeps moving to the outer death point, and the compression piston remains stationary at the central death point. As the volume increases, due to the incorporation of heat (Q_E) to the system from an external source, the pressure decreases at a constant temperature.

Process 4-1: Another transfer process describes the closure of the cycle, as in process 2-3 the pistons move simultaneously enclose the same volume and transfers the fluid back to the compression space but passing before through the regenerator. Therefore, a change in temperature occurs as before but in the opposite direction which means T_{max} to T_{min}.

Schmidt analysis

The Schmidt cycle is the most complicated case to find an analytical solution; all the remaining studies must be solved numerically due to their restrictive assumptions. It gets its name after Gustav Schmidt, who published this analysis in 1871.

The analysis is based on mainly five assumptions:

- The working fluid obeys the ideal gas law
- The mass inside the engine is constant
- The instantaneous pressure is consistent within the working space
- There are isothermal regions throughout the engine
- The piston and displacer motions behave sinusoidally

Eq. (1) shows the pressure as a function of the compression (V_C) and expansion (V_E) volumes.

$$P = \frac{mR}{\frac{V_C}{T_C} + \frac{V_k}{T_C} + \frac{V_r \ln(T_E/T_C)}{T_E - T_C} + \frac{V_h}{T_E} + \frac{V_E}{T_E}} \quad (1)$$

Where:

V_C = Compression volume
V_K = Cooler volume
V_r = Regenerator volume
V_h = Heater volume
V_E = Expansion volume
T_C = Compression temperature
T_E = Expansion temperature
P = Pressure
R = Gas constant
M = Total mass inside the engine

From assumption 5, the volumes are periodic function; thus, the work done per cycle W is shown in Eq. (2) and the integration is carried out over the common period of the volumes.

$$W = mR\oint \frac{d(V_E + V_C)}{\frac{V_C}{T_C} + \frac{V_k}{T_C} + \frac{V_r \ln(T_E/T_C)}{T_E - T_C} + \frac{V_h}{T_E} + \frac{V_E}{T_E}} \quad (2)$$

Figure 3 shows the temperature variation within the regenerator T_r, which is determined by Eq. (3). Also, the temperature in the compression space and cooler space is the same due to the assumption of isothermal regions. Similarly, the heater space and expansion space have the same temperature.

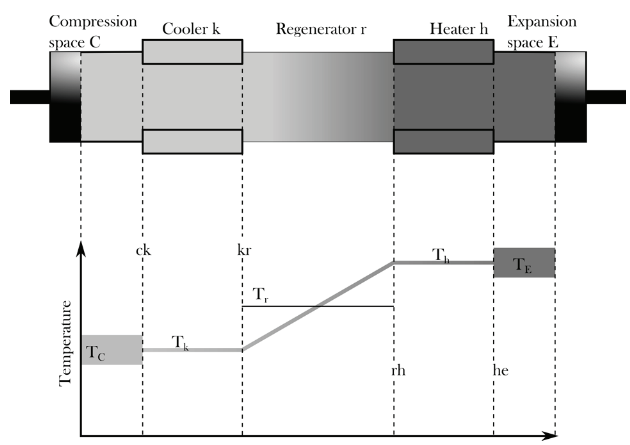

Figure 3. Isothermal model (Schmidt analysis)

$$T_r = \frac{T_E - T_C}{\ln\left(\frac{T_E}{T_C}\right)} \quad (3)$$

Dynamics of free piston Stirling engine

The study of Stirling engines is a multidisciplinary task. It includes knowledge from solid mechanics, fluid mechanics, and heat transfer. Since the difference in temperatures and the motion of the pistons within the cylinder together determine the pressure variation, the analysis should be studied as a whole and not only the solid mechanic part but the thermodynamic phenomena.

FPSEs are more complicated due to their multiple degrees of freedom (DOF) compared to standard inside combustion engines. They are constituted by three masses, although in many cases, the piston or cylinder is fixed to the ground, so the three DOF reduces to two. Additionally, various springs and dashpots connect the moving devices.

Figure 4, shows two masses: one for the piston (P) and the other for the displacer (D), three dashpots and two springs of stiffness K_D and K_P, two sinusoidal external forces F_D and F_P of the same frequency and in phase but with different amplitudes which are applied to D and P respectively.

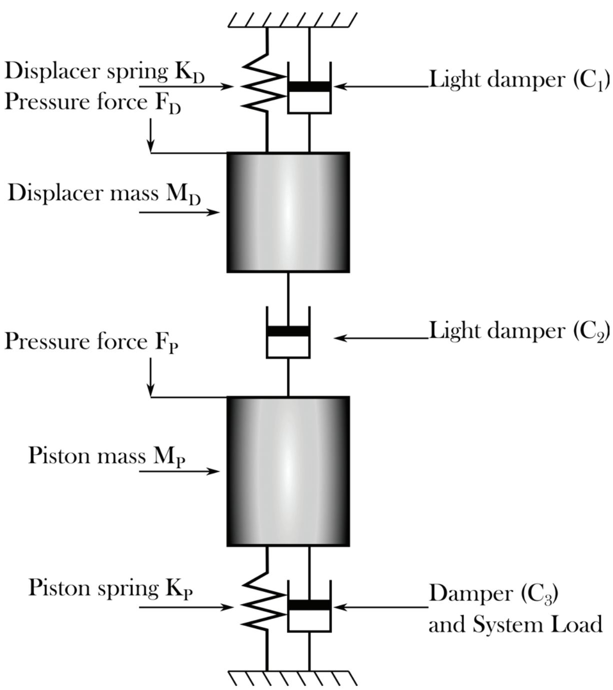

Figure 4. Piston and displacer diagram

Equations of motion

Because FPSE lack of mechanical connections, the analysis of this type of machines must have a pre-study before applying the first-order analysis. The most significant difference between the FPSE and the kinematic engines is that the piston and displacer strokes are unspecified, as well as the phase angle. Not only the fact that the amplitudes are unknown is hard to lead but also the dependence of these parameters with loads adds more complexity to the study. Therefore, the study of the dynamics is fundamental and thus it is the first step.

The approach used in this section is based on variational calculus, attributed to Leibniz and Lagrange and it is referred to as analytical dynamics. It considers the system as a whole rather than its individual components, two fundamental quantities, the kinetic energy, and work, both of which are scalar quantities. Since the purpose of the document is to analyze the engine, the demonstration of the method is not covered. Hence the study begins with the Lagrangian equation (Meirovitch, 1998).

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}-\frac{\partial L}{\partial q_k}=Q_k \tag{4}$$

$$Q_k=\sum\nolimits_{j=1}^{p}\vec{F}_J\cdot\frac{\partial \vec{r}_J}{\partial q_k} \tag{5}$$

Where L is the Lagrangian and q_k are the generalized coordinates. The kinetic energy of the system is divided into the displacer kinetic energy ($\tau_{Displacer}$) and the piston kinetic energy (τ_{Piston}). See Eqs. (6) and (7).

$$\tau_{Displacer}=\frac{1}{2}M_D\cdot\dot{\vec{r}}_1^{\,2} \tag{6}$$

$$\tau_{Piston}=\frac{1}{2}M_P\cdot\dot{\vec{r}}_2^{\,2} \tag{7}$$

Where r_k are the dependent variables. The total kinetic energy (τ_{Total}) is described in Eq. (8):

$$\tau_{Total}=\frac{1}{2}\left(M_D\cdot\dot{\vec{r}}_1^{\,2}+M_P\cdot\dot{\vec{r}}_2^{\,2}\right) \tag{8}$$

The potential energy of the system is divided into the displacer potential energy ($\nu_{Displacer}$) and the piston potential energy (ν_{Piston}). See eq. (9) and (10).

$$\nu_{Displacer}=\frac{1}{2}\left(k_D\vec{r}_1^{\,2}+k_R(\vec{r}_2-\vec{r}_1)^2\right) \tag{9}$$

$$v_{Piston} = \frac{1}{2}\left(-k_P\vec{r}_2^{\,2} + k_R(\vec{r}_2 - \vec{r}_1)^2\right) \tag{10}$$

The total potential energy (v_{total}) is described in eq. (11).

$$v_{Total} = \frac{1}{2}\left(k_D\vec{r}_1^{\,2} + k_R(\vec{r}_2 - \vec{r}_1)^2 + k_R(\vec{r}_2 - \vec{r}_1)^2 - k_P\vec{r}_2^{\,2}\right) \tag{11}$$

The non-conservative forces described on eq. (5) are also divided into two, thus there are only two degrees of freedom. First, the non-conservative forces for the displacer are shown in Eqs. (13) - (15).

$$Q_1 = \sum\nolimits_{j=1}^{3} \vec{F}_J \cdot \frac{\partial \vec{r}_J}{\partial q_1} \tag{12}$$

$$\vec{F}_1 = -c_D\vec{r}_1 \cdot 1 \tag{13}$$

$$\vec{F}_2 = -c_R(\vec{r}_2 - \vec{r}_1) \cdot 1 \tag{14}$$

$$\vec{F}_3 = \Delta P \cdot A_c \tag{15}$$

Similarly, the non-potential forces acting on the piston are described in Eqs. (17-19).

$$Q_2 = \sum\nolimits_{j=1}^{3} \vec{F}_J \cdot \frac{\partial \vec{r}_J}{\partial q_2} \tag{16}$$

$$\vec{F}_1 = -c_P\vec{r}_2 \cdot 1 \tag{17}$$

$$\vec{F}_2 = c_R(\vec{r}_2 - \vec{r}_1) \cdot 1 \tag{18}$$

$$\vec{F}_3 = \Delta P \cdot (A_c - A_r) \tag{19}$$

Finally, Eq. (4) is used in order to find the equations of motion:

$$m_D\ddot{x} + (c_D + c_R)\dot{x} + (k_D + k_R)x - c_R\dot{y} - k_R y = \Delta P \cdot A_r \tag{20}$$

$$m_P\ddot{y} + (c_P + c_R)\dot{y} + (k_P + k_R)y - c_R\dot{x} - k_R x = \Delta P \cdot (A_c - A_r) \tag{21}$$

Where, $x = r_1$ and $y = r_2$ which are the two DOF and ΔP, A_r and A are the variation in pressure, the rod area and the cylinder area, respectively.

Stability analysis of free piston Stirling engines

A stability analysis is presented as an essential step to study how different design parameters influence the behavior of the thermomechanical machine (Kwankaomeng *et al.*, 2014).

The section starts with the new dynamic model, which includes a new velocity-dependent mechanical loss. The latter is included with the purpose of studying how the motion of the pistons influence each other. The mathematical tool used is the Laplace transform, due to its simplicity of implementation and above all for the study of the localization of the poles.

The main obstacle to designing functional Stirling engines is to guarantee the stability of the oscillations, which is not a trivial task due to the decoupled pistons and the sensibility of some parameters. There is a list of tips presented in (Walker, 1980) although most of them are assuming the construction of the machine.

Based on the analysis made by Redlich and Berchowit (1985), a more complete dynamic system described in Eqs. (20) and (21) is now studied. The spring constant between the displacer and the piston is assumed negligible in comparison to the mechanical piston spring constant and the mechanical displacer spring constant.

$$c_D + c_R = D_D \tag{22}$$

$$k_D + A_r\,\partial P / \partial x_D = K_D \tag{23}$$

$$c_P + c_R = D_P \tag{24}$$

$$k_P + A_P\,\partial P / \partial x_P = K_P \tag{25}$$

$$c_R S - A_r\,\partial P / \partial x_P = \varpi_P\,(S) \tag{26}$$

$$c_R S - A_P\,\partial P / \partial x_D = \varpi_D\,(S) \tag{27}$$

Eqs. (28) and (29) are deduced.

$$(m_D s^2 + D_D s + K_D)\,X_D = \varpi_P(s)\,X_P \tag{28}$$

$$(m_P s^2 + D_P s + K_P)\,X_P = \varpi_D(s)\,X_D \tag{29}$$

Important parameters of the system are the frequency and the dynamic losses (Redlich & Berchowit, 1985), Eqs. (30) and (31).

$$\omega_D = \sqrt{\frac{k_D}{m_D}}, \quad Q_D = \frac{\omega_D m_D}{2\pi D_D} \tag{30}$$

$$\omega_P = \sqrt{\frac{k_P}{m_P}}, \quad Q_P = \frac{\omega_P m_P}{2\pi D_P} \tag{31}$$

$$H_D(s) = m_D\left(s^2 + \frac{\omega_D}{2\pi Q_D} + \omega_D^2\right) \tag{32}$$

$$H_P(s) = m_P\left(s^2 + \frac{\omega_P}{2\pi Q_P} + \omega_P^2\right) \tag{33}$$

The poles of the system are calculated from Eqs. (32) and (33), which are the roots of the polynomial.

$$H_D(s) = 0 = -\frac{1}{2}\left(\frac{\omega_D}{2\pi Q_D} \pm \sqrt{\left(\frac{\omega_D}{2\pi Q_D}\right)^2 - 4\omega_D^2}\right) \tag{34}$$

$$H_P(s) = 0 = -\frac{1}{2}\left(\frac{\omega_P}{2\pi Q_P} \pm \sqrt{\left(\frac{\omega_P}{2\pi Q_P}\right)^2 - 4\omega_P^2}\right) \tag{35}$$

Depending on the localization of the poles ω_c can be calculated from Eq. (36). Figure 5 graphically shows the values of the roots and how these influence the value of ω_c.

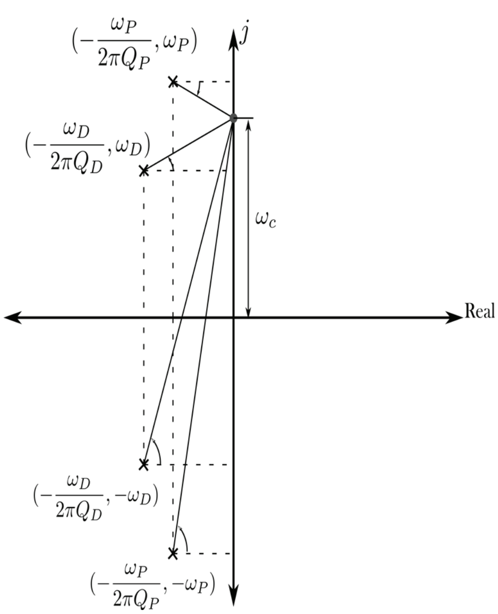

Figure 5. Pole location in the complex plane

$$\omega_c = \omega_D\, \omega_P\, (Q_D + Q_P) \,/\, \omega_P\, Q_P + \omega_D\, Q_P \tag{36}$$

Redlich determined that the next criteria must be fulfilled in order to maintain oscillations within the cylinders (Eq. 37).

$$|H_D(j\omega_c)\, H_P(j\omega_c)| < |\varpi_P(j\omega_c)\, \overline{\varpi}_D(j\omega_c)| \tag{37}$$

Thermodynamic study

Beta type machines differ from the alpha type due to the compression space, which depends on the displacer and piston motion. Eqs. (38) and (39) describes both volumes (Urieli, 1984a).

$$V_C = V_{clc} + V_{swc}\,(1 + \cos(\theta + \delta))\,/\,2 \tag{38}$$

$$V_E = V_{cle} + V_{swe}\,(1 + \cos(\theta + \delta + \alpha))\,/\,2 \tag{39}$$

Where: $\alpha = \pi + \phi - \delta$

Later in the analysis, the variation of the compression and expansion volumes with respect to the cycle will be used to compute the work done and thus the power.

$$P = \frac{mR}{S + \frac{V_{swc}}{2T_C}\cos(\theta+\delta) + \frac{V_{swe}}{2T_E}\cos(\theta+\delta+\alpha)} \tag{40}$$

Urieli (1984b) presented a mathematical technic to simplify the explicit pressure form in eq. (40).

$$S = \frac{V_{clc}}{T_C} + \frac{V_{swc}}{2T_C} + \frac{V_k}{T_C} + \frac{V_{cle}}{T_E} + \frac{V_{swe}}{2T_E} + \frac{V_h}{T_E} + \frac{V_r \ln(T_E/T_C)}{T_E - T_C} \tag{41}$$

$$\psi = \delta + \alpha \tag{42}$$

After applying some trigonometric identities and rearranging some terms, the result is Eq. (43).

$$P = \frac{mR}{S + \left(\frac{Vswc}{2T_C}\cos\delta + \frac{Vswe}{2T_E}\cos\psi\right)\cos\theta - \left(\frac{Vswc}{2T_C}\sin\delta + \frac{Vswe}{2T_E}\sin\psi\right)\sin\theta} \tag{43}$$

New equalities to simplify Eq. (43). The graphic representation of the angle (β) and the hypotenuse (c) are also shown Figure 6.

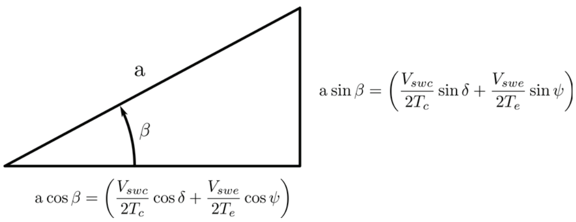

Figure 6. Trigonometric representation of volumes

$$\tan\beta = \frac{\frac{V_{swc}}{2T_C}\sin\delta + \frac{V_{swe}}{2T_E}\sin\psi}{\frac{V_{swc}}{2T_C}\cos\delta + \frac{V_{swe}}{2T_E}\cos\psi} \tag{44}$$

$$C = \sqrt{\left(\frac{V_{swc}}{2T_C}\sin\delta + \frac{V_{swe}}{2T_E}\sin\psi\right)^2 + \left(\frac{V_{swc}}{2T_C}\cos\delta + \frac{V_{swe}}{2T_E}\cos\psi\right)^2} \tag{45}$$

Substituting Eqs. (44) and (45) in eq. (43):

$$P = \frac{mR}{S + c\cos(\beta + \theta)} \tag{46}$$

Then, the work of the engine can be calculated from eq. (47) and (48).

$$Q_C = W_C = -\frac{mRV_{swc}}{2}\int_0^{2\pi}\frac{\sin(\theta+\delta)}{S + c\cos(\theta+\beta)}d\theta \tag{47}$$

$$Q_E = W_E = -\frac{mRV_{swe}}{2}\int_0^{2\pi}\frac{\sin(\theta+\delta+\alpha)}{S + c\cos(\theta+\beta)}d\theta \tag{48}$$

From eq. (45) the total work is calculated.

$$W = W_E + W_C \tag{49}$$

Design parameters for a 1kW FPSE

Evolution has played the primary role in nature, the mixing and mutation of genes create new combinations in the DNA, despite the similar characteristics, a new offspring can be better than its predecessor. This is the foundation for the Genetic Algorithm.

The algorithm tries to mimic the logic of nature; the more fitted chromosomes are allowed to "reproduce" their genes to form a next and probably a better generation and the less capable "died" then the new generation repeat the cycle, so the enhancement of the species continues, on this case till the convergence of a solution. The implementation of the algorithm simplifies the natural evolution in five steps (Mallawaarachchi, 2017). See Figure 7 for a better understanding of the algorithm:

- Initial population: To begin with the algorithm it is necessary to create the first generation of individuals, each of them is a particular solution for the problem tended to solve (not necessarily a good solution). These individuals are described by a set of parameters named genes that are equivalent to variables; a collection of genes creates a chromosome that is a solution.
- Fitness function: A fitness function determines how capable an individual compares to the rest of the population; each is graded, and this will determine the availability to reproduce itself.
- Selection: As natural selection determines how a population could grow and the fittest individual can pass their genes to the next generation, something similar occurs with the population in the algorithm, based on their fitness function score they either are killed or allow to reproduce.
- Crossover: Crossover is used to mix some genes from their parents to a couple of newborn individuals with new genes and probably with better capabilities, in the algorithm how genes mix can be defined either arbitrary or in a deterministic manner.
- Mutation: Stagnation in the community can be prevented using a method called mutation, although it is used randomly also is found to be helpful. It consists of modifying specific genes with a different value thus it enhances a significant diversity in the population.
- Based on the five steps mentioned earlier, a code is developed with the purpose of pursuing a set of parameters, which enables the engine to generate a power output of 1 kW.

Results and discussion

The results shown in Table 1 represent the implementation of the first-order analysis for a beta type engine, based on Urieli's work, and the stability analysis considering the loss between the power piston and the displacer. The error presented compares the first order analysis and the stability study developed with the Schmidt analysis deduced from Urieli (1984a).

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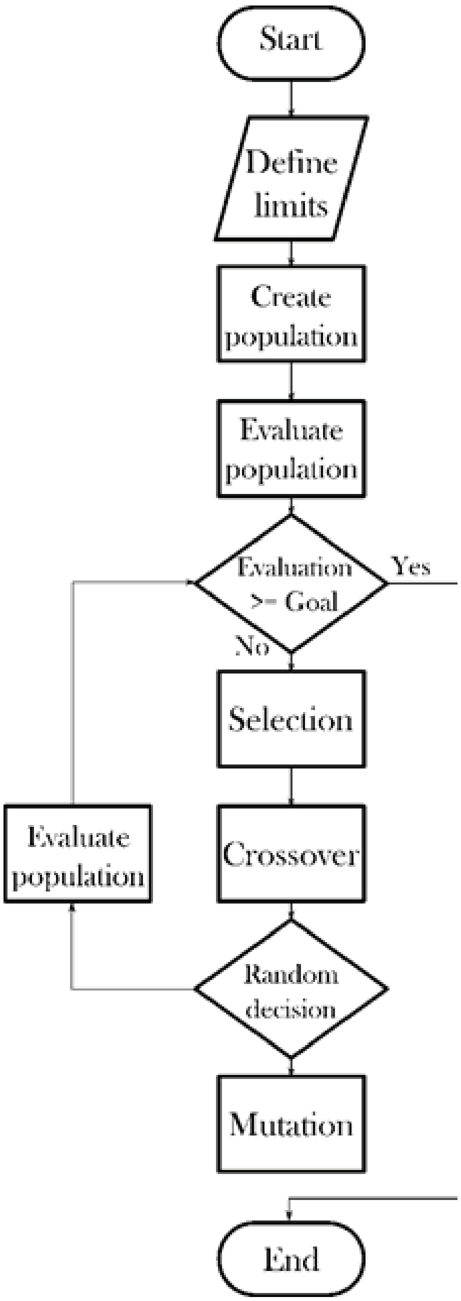

Figure 7. Flow diagram for the search of parameters

Table 1. Design parameters for a 1kW FPSE

Núm.	Cyl. area (m^2)	Rod area (m^2)	Charge press. (Bar)	Cooler Temp. (K)	Heater Temp. (K)	Piston mass (kg)	Displacer mass (kg)	Piston spring stiff-ness (N/m)	Displacer spring stiff-ness (N/m)	Freq. (Hz)	Phase angle (Rad)	Power (W)	Error (%)
1	0.0091	0.0016	6.758464	322.2406	1116.68	1.0544	0.2541	1969.9066	1919.6185	9.239	1.9371	1010.87	-0.05
2	0.0204	0.0008	6.577653	439.3334	981.397	2.1052	1.6476	675.6721	1011.0944	2.415	1.9431	1064.22	0.3173
3	0.0124	0.0008	6.874775	489.0427	1136.18	2.3059	0.6414	906.4282	1830.083	4.934	1.7558	868.51	0.5928
4	0.0225	0.003	5.71042	492.3117	900.127	1.045	1.3351	576.5377	1375.2158	2.893	1.722	1015.95	-12.48
5	0.009	0.0003	6.878591	335.8463	1057.27	2.5074	0.2277	852.8553	1931.3532	8.343	1.6572	1010.62	0.6071
6	0.0249	0.0027	1.540996	303.4345	1018.95	2.6616	0.3165	544.3997	1277.6503	5.722	1.6459	989.201	-0.691
7	0.0273	0.0066	1.705393	397.7975	1152.47	2.9107	0.3609	583.056	988.0791	4.985	1.716	997.08	-4.658
8	0.0138	0.0038	4.51109	439.5299	973.728	1.5836	0.1942	894.4902	1730.9642	8.785	1.6801	1003.82	0.8297
9	0.0249	0.0058	3.033995	380.244	916.188	0.6708	0.8787	581.1874	1544.6524	2.619	1.9661	769.969	2.5703
10	0.011	0.0054	5.094746	448.0015	1171.28	2.0799	0.1043	2026.1392	1739.172	14.12	1.8363	1001.99	-0.02
11	0.0201	0.0026	5.360449	403.1546	1074.39	1.0112	1.3884	304.6936	1622.2275	1.448	1.7892	525.892	-0.443
12	0.0299	0.0028	1.036066	369.4373	828.76	1.4359	0.4718	1375.3947	1210.4832	5.97	1.9334	983.189	-12.31
13	0.0176	0.0076	6.80081	357.5295	669.773	1.1283	0.3908	385.0678	1203.0731	4.779	1.5926	997.921	0.7554
14	0.0158	0.0037	2.420829	420.3085	1138.66	2.9003	0.8034	834.2437	1293.708	3.886	1.854	368.62	0.0331
15	0.0362	0.0071	1.330347	319.6724	1109.9	1.417	0.8711	690.3573	792.5066	3.25	1.91	1042.68	1.0819
16	0.0181	0.0062	6.00861	300.4698	1143.28	0.9532	0.5657	350.0767	599.9818	3.096	1.7097	998.62	0.5184
17	0.023	0.0002	3.992044	496.8031	764.759	2.5184	1.0459	851.0623	1154.4598	3.301	1.94	988.495	2.6773
18	0.0158	0.0001	3.009427	345.7366	1159.32	1.5142	0.2699	209.7209	1964.409	6.428	1.4571	989.627	2.3484
19	0.0103	0.0006	5.262286	312.2817	1024.37	1.503	0.1273	391.0977	1223.0295	8.441	1.5832	1003.96	0.865
20	0.0173	0.0023	5.634158	426.0448	1030.97	1.4131	0.7294	625.3536	1120.9107	3.697	1.7678	998.789	0.1179

Table 1 shows twenty different combinations of parameters, which represent twenty different engines working in a stable mode, and with power outputs closer to the desired 1 kW. There are also some designs with lower power outputs, like the engines 11, and engine 14. This indicates that under certain conditions, there is a maximum limit for the power output that would not affect the stable operation of the engine. In the case of engines 11 and 14, this limit is 525 W, and 368.62 W respectively.

The proposed procedure doesn't explore in detail the influence of a particular parameter, the objective is to search through all the potential designs, ensuring the stability of each proposed engine. This represents a valuable tool for early-stage design of FPSE.

In general, the error for the predicted power is lower than 1 % for the majority of the simulations, which validates the proposed models.

Figures 8 and 9 show the variation of the volume inside the compression and expansion spaces for engines 3, and 12, a pair of engines enlisted in Table 1. Pressure-Volume diagrams for the mentioned engines are presented in Figures 10 and 11.

Figures 10 and 11 reflect the characteristic shape for a PV diagram in Stirling engines. This shape differs largely from the theoretical PV cycle and thus indicates the relevance of using adequate tools to analyze Stirling cycles.

Some conclusions can be inferred from this kind of diagram. For example, according to Figures 8 and 9, both engines follow a similar volume variation, but the design 3 gives a higher power output. This difference can be explained due to the operation at higher pressures observed in Figure 10 when compared with the pressure levels of engine 12, Figure 11. This supports a known fact about Stirling engines: when two engines present similar geometrical characteristics, the operation at higher pressures translates into higher power outputs.

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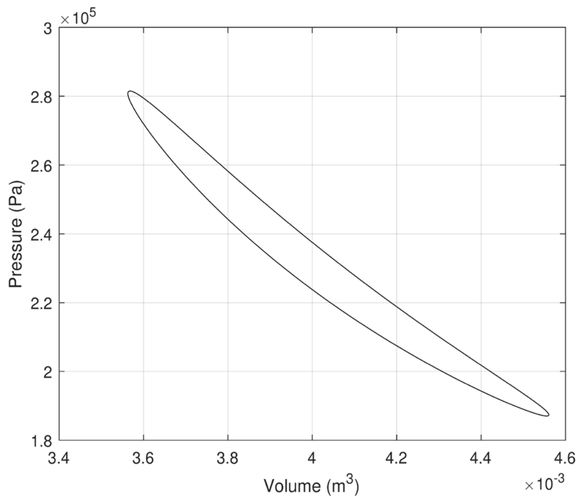

Figure 8. Variation of the volumes (Engine 3)

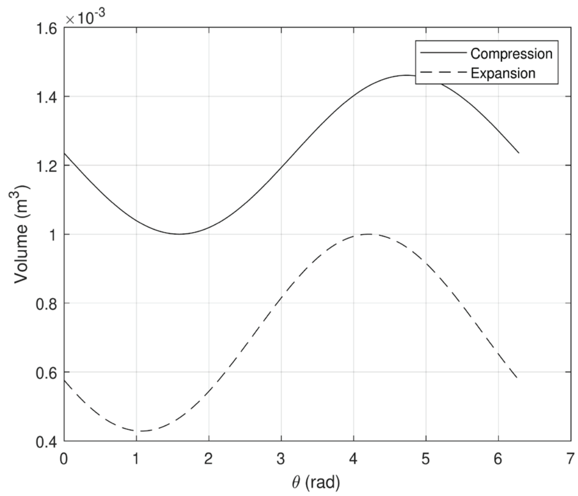

Figure 10. PV Diagram (Engine 3)

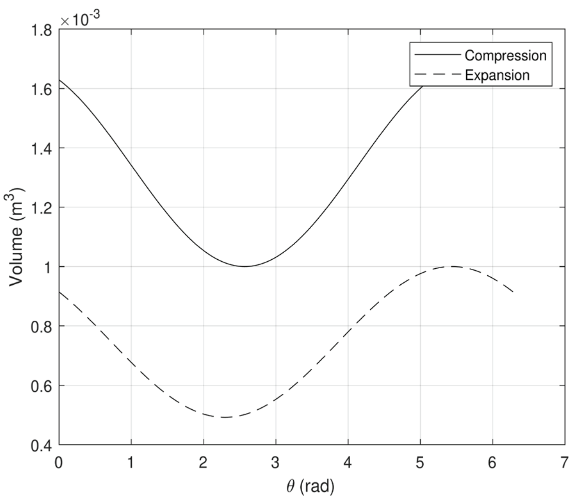

Figure 9. Variation of the volumes (Engine 12)

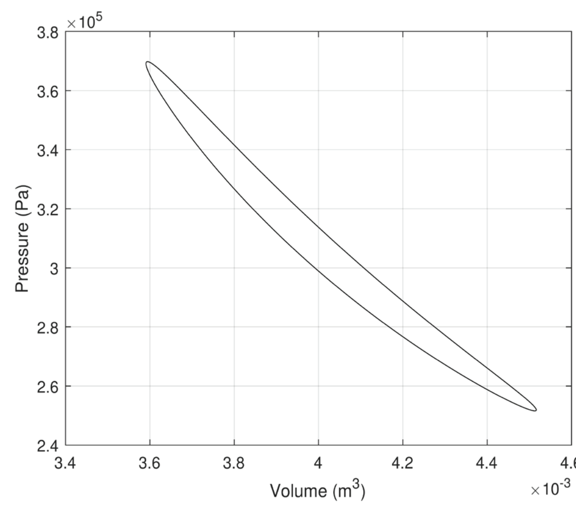

Figure 11. PV Diagram (Engine 12)

Conclusions

The combination of stability, dynamic restraints and equation of motions for Free Piston Stirling Engines has been achieved in this work.

Functional designs were obtained assuring the stability of the engine. FPSE results more complicated than the kinetic engines due to the uncoupled pistons, which lead to unknown movements. Therefore, it is crucial for the study to solve the equations of motion first. A small variation in the parameters such as mass and spring stiffness could strongly influence the performance or the proper functioning of the engine.

This work showed that it is critical to consider this sensibility of the system, and consequently, it is essential to apply a stability analysis in order to fully understand the behavior of the thermal machine. Furthermore, it was proved that a genetic search algorithm is a viable method to find a desirable power output.

Nomenclature

A_c = Area of the cylinder
A_r = Area of the displacer rod
F = Forces
T_k = Cold temperature

T_h = Hot temperature
T_D = Regenerator temperature
V_k = Compression volume
V_h = Expansion volume
V_D = Death volume
R = Ideal gas constant
m = gas mass
k_D = Displacer spring stiffness
k_P = Piston spring stiffness
k_r = spring stiffness
c_D = Displacer viscous coefficient
c_P = Piston viscous coefficient
c_R = viscous coefficient
L = Lagrangian
m_D = Displacer mass
m_P = Piston mass
ω = Frequency
ϕ = Phase angle
λ = Eigenvalues
P = Pressure
P_0 = Initial pressure
P_a = Ambient pressure
P_m = Mechanical power
q = Generalized coordinate
r_r = Rod radio
r_c = Bore radio
$\tau_{Displacer}$ = Displacer kinetic energy
τ_{Piston} = Piston kinetic energy
τ_{Total} = Total kinetic energy
$\nu_{Displacer}$ = Displacer potencial energy
ν_{Piston} = Piston potencial energy
ν_{Total} = Total potencial energy

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