Ingeniería Investigación y Tecnología
Volume XXVI (Issue 4), October-December 2025 1-16
ISSN 2594-0732 FI-UNAM Peer-reviewed article
Article information: Received: February 18, 2025, reevaluated: May 30, 2025, accepted: August 29, 2025.

https://doi.org/10.22201/fi.25940732e.2025.26.4.029

Harmonics signal analysis with noise through orthogonal vector base decomposition method

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Abstract

This paper introduces an innovative method for studying dynamic electrical signals through orthogonal base vector decomposition. This approach effectively harnesses the properties of an orthogonal basis, even amid noise, utilizing a generalized Fourier series framework to enhance understanding and accuracy. Central to this methodology is the principal component function, which enables the decomposition of pseudo-random noise and error functions based on both the correlation index and generalized Fourier approximated series. This paper presents compelling test applications that analyze synthetic harmonic and interharmonic signals, as well as a real measured signal using Chebyshev orthogonal polynomials, Legendre polynomial approximations, and Fourier series approximations. This method not only proves to be efficient with remarkable accuracy but also maintains a low computational burden, making it highly practical for real-world applications. By employing the correlation coefficient of the polynomials and function approximations, we provide a qualitative comparison that underscores the accuracy of signal synthesis against the traditional Fourier theory application. In conclusion, the insights and characteristics derived from this research empower the extraction of crucial modal component information on the propagation, attenuation, and velocities of the electrical signals under investigation. This advancement holds significant promise for future applications in the field of electrical engineering.
Keywords: Orthogonal, vector, decomposition, harmonics, Chebyshev polynomial, Legendre functions, Fourier series.

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Introduction

The new current trends in electrical network technology require for monitoring, control and protection of devices based on power electronics, which has significantly increased harmonic pollution and power quality problems in the distribution network up to power levels (Taylor, 1999; Rehtanz, 2025; Taranto *et al.*, 2025).

This in turn, paradoxically, has also increased losses in distribution systems due to overheating of motors and transformers, malfunctions such as false tripping of protective equipment such as relays and circuit breakers, and even producing severe direct distortion of the electrical waveform (Plet, 2024).

Due to the technical relevance of the harmonic pollution problem worldwide, several international working groups have focused their efforts to understand the behavior of this phenomenon through its different stages of detection, measurement, control and elimination of dynamic electrical signals propagating in electrical systems (European Standard EN50160, 1999; The Norwegian Directive on Quality of Supply, 2005; CIGRE WG. C4.07 Power quality indices and objectives, 2004; Lines, 1999; IEEE 519-1992, 1993). As a result, several approaches based on classical Fourier and other signal processing-based techniques have been published in the technical literature which in part has motivated to improve the identification, characterization and diagnosis of this particular problem (Arrillaga, 2003; Ribeiro *et al.*, 2014; Bollen & Gu, 2006).

The increasing harmonic and interharmonic operating conditions of power systems cause current and voltage waveforms to continuously distort over time. Consequently, the applicability of Fourier-based tools is limited to a narrow range of power quality studies (Arrillaga, 2003; Ribeiro *et al.*, 2014; Bollen & Gu, 2006). However, in electrical engineering, it is the cornerstone of many deterministic and heuristic techniques for linear systems and signal processing applications (Lathi, 2005; Fourier, 1822; Uribe, 2024).

To overcome the limitations of Fourier-based tools, different orthogonal bases have been used, which have given rise to the generalized Fourier series (Lathi, 2005). One of the advantages of this proposed methodology is the possibility of extending the Fourier-based tools to a non-harmonic or interharmonic synthesis, different from the classical orthogonal basis formed only by stationary trigonometric functions (sines and cosines). To study this option, there are a large number of signal sets that can be used as orthogonal bases to form the generalized Fourier series (Lathi, 2005; Fourier, 1822; Abramowitz & Stegun, 1965; Zhang & Jin, 1996; MATLAB Version: 9.13.0.2049777 (R2022b), 2025). In particular, in Mason & Handscomb (2003) and Cai & Tak (2004) it has been shown that the Legendre and Chebyshev approximations are almost identical to the optimal minimax polynomial (which is the best possible polynomial to minimize the maximum deviation from the real signal among all existing polynomials of the same order). In addition, Chebyshev polynomials have an advantage over Legendre polynomials because the positions of their zeros are known analytically and are easier to calculate. This mathematical advantage opens new possibilities of potential applications for the realization of signal synthesis with low complexity or computational effort. This represents a relevant feature that allows extending this technique for advanced diagnostics and harmonic control of dynamic equipment such as wind generators, solar panels or motor vehicle batteries that use power electronics.

Finally, the main contribution of this article is to present an analysis method for testing real electrical noisy signals obtained by direct measurement with digital phosphor oscilloscope from the properties of the orthogonal vectors of a generalized function or series based on Fourier, Chebyshev and Legendre polynomials.

Fundamentals of Fourier Theory

A sinusoidal electrical signal with frequency ω_0 magnitude C and phase θ can be decomposed into the following two sinusoidal signals of the same frequency as shown in Figure 1a (Lathi, 2005):

$$C\cos(\omega_0 t+\theta)=a\cos\omega_0 t+b\sin\omega_0 t \tag{1}$$

Where $\omega_0 = 2\pi f$ is the angular frequency (f), t is time domain variable and the coefficients are $a = \cos\theta$ and $b = -\sin\theta$.

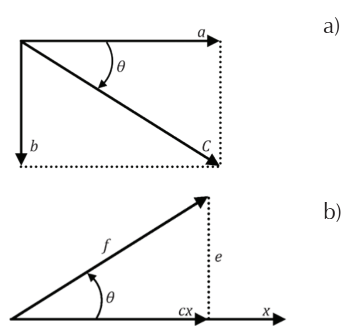

Figure 1. a) Two sinusoids additive property, b) Vector projection with error component (Lathi, 2005).

The Fourier series can be obtained extending the analysis for "n" vector components on both left- and right-hand sides of (1) having:

$$\sum_{n=0}^{\infty} C_n\cos(n\omega_0 t+\theta_n)=\sum_{n=1}^{\infty} a_n\cos(\omega_0 t)+b_n\sin(\omega_0 t) \tag{2}$$

Where the trigonometric coefficients of (2) are easily calculated with:

$$C_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) \tag{3}$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos(n\omega_0 t)dt \tag{4}$$

and

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin(n\omega_0 t)dt \tag{5}$$

for $n = 1, 2, 3, ...$
Approximation (2) is known as the Fourier synthesis or in the case of the left hand-side as the compact trigonometric series arrangement for a complete period of the signal (Fourier, 1822; Lathi, 2005):

$$f(t) \approx C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) \tag{6}$$

Where:

$$C_0 = a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)dt \tag{7}$$

and $T_0 = 2\pi / \omega_0$.

Signal comparison fourier projection coefficients

When a real signal or a vector index comparison between other two vectors is required, or as in the case of evaluating the magnitude of two vectors, the correlation coefficient can be effectively used. Consider the following relation for approximating vector f in cx as shown in Figure 1b with a considerable error projection of e (Lathi, 2005):

$$f \cong cx + e \tag{8}$$

and considering that when the error signal, or the vector projection along axis x, is the smallest, in this sense, we can substitute the value of the cosine function between both vectors as follows (Lathi, 2005):

$$c|x| = |f|\cos\theta \tag{9}$$

Where c is a scalar, and $|x|$ is the vector length and $\cos\theta$ is the angular function between both vectors. By multiplying both sides of (9) by the magnitude of x, we can identify the correlation coefficient as follows (Lathi, 2005):

$$|f|\,|x|\cos\theta = c|x|^2 = f \cdot x \tag{10}$$

or

$$c = \frac{f \cdot x}{x \cdot x} = \frac{1}{|x|^2} f \cdot x \tag{11}$$

Where the inner product for the real signal case is defined by $f(t) \cdot x(t) = \int_{t_1}^{t_2} f(t)x(t)dt$ inside the interval (t_1, t_2). If we consider that the vector x can be approximated with function $\cos(n\omega_0 t)$, then we obtain the following expression:

$$c_n = \frac{\int_{t_1}^{t_2} f(t)\cos(n\omega_0 t)dx}{\int_{t_1}^{t_2} \cos^2(n\omega_0 t)dx} = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos(n\omega_0 t)dt \tag{12}$$

which is the corresponding coefficient in (4) of (2) for $n = 1, 2, 3$ ... Similary, the corresponding coefficient (5) of (2) is obtained in the case of $x \cong sin\,(n\omega_0 t)$ is approximated. Which is the case of the electrical signals when forming an stationary base related to the Fourier theory (Arrillaga, 2003).

Normalized correlation property

In the previous subsection, Eq. (11) shows that c can be considered as a quantitative measure of similarity between f and x. Or as a signal comparison expression between two signals. This similarity measure c is better known as the non-normalized correlation coefficient and, from (10), it observes that $c = \cos(\theta)$, is ranging between, but expression (11) can be normlized with respect the unitary value of magnitude if we add the term $1/\sqrt{E_X}$ in the denominator of (11):

$$-1 \le c \le 1 \tag{13}$$

By using the same argument in defining a similarity index for signals, a more general criterion based in c independent of the energy (size) of $f(t)$ and $x(t)$ is given by, (Lathi, 2005):

$$c = \frac{1}{\sqrt{E_f E_x}}\int_{-\infty}^{\infty} f(t)x(t)dt \tag{14}$$

Where $E_f = \int_{-\infty}^{\infty} f^2(t)\,dt$ is the energy of $f(t)$, and E_x is the correponding expression to $x(t)$, and can be extended to the entire time interval from $-\infty$ to ∞.

Orthogonal properties of generalized-fourier

To perform a numerical analysis of a signal there is a large number of orthogonal signal sets which can be used as a reference basis of projection for another signals better known as the generalized Fourier-series (Lathi, 2005; Fourier, 1822). Some of the most often used signal sets to approximate signals in electrical engineering applications are the sinusoid or exponential functions. Other kinds of signal sets, less used in the electrical engineering area are based on algebraic polynomial extensions or on direct function approximations or by using interpolants. Some of these orthogonal polynomial bases are Chebyshev, Laguerre, Jacobi and Hermite. To approximate functions we also have Legendre, Walsh, Bessel and Kelvin series solution (Bollen & Gu, 2006; Lathi, 2005; Fourier, 1822; Abramowitz & Stegun, 1965; Zhang & Jin, 1996).

In Mason & Handscomb (2003) it has been shown that the Chebyshev approximation is almost identical to the optimal minimax polynomial problem which requires low computational cost than another traditional method. Indeed, in (Mason & Handscomb, 2003) the Chebyshev polynomials have been used as a basis for approximating and indexing *d*-dimensional trajectories of different type of signal applications.

Thus, a methodology based on the orthogonal properties of one of the base Fourier generalized series form can be used to explore the signal properties for the analysis of electrical engineering harmonic and inter-harmonic signals. For the analysis presented in this paper an application of the first and second order of Chebyshev function polynomials and the Legendre approximate functions has been implemented in a MATLAB platform in a generic PC (MATLAB Version: 9.13.0.2049777 (R2022b), 2025).

A- Chebyshev polynomial of the first kind

The Chebyshev polynomial of the first kind is described in Abramowitz & Stegun (1965) and Zhang & Jin (1996) as:

$$T_n(x) = \cos(n \cdot \cos^{-1} x) \tag{15}$$

for all $x \in [-1, 1]$.

As an example, for three successive orthogonal polynomials the recurrence relations are:

$$T_0(x) = 1,\ T_1(x) = x$$

and so, on:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \tag{16}$$

Moreover, the Chebyshev polynomials of the first kind are orthogonal on [−1, 1], with respect to the weight linear function $w(x) = 1/\sqrt{1-x^2}$, if their inner product satisfies:

$$T_m(x) \cdot T_n(x) = \int_{-1}^{1} w(x) T_m(x) T_n(x) dx$$

Where we obtain:

$$= \begin{cases} \pi & m = n = 0 \\ \dfrac{\pi}{2} & m = n \neq 0 \\ 0 & m \neq n \end{cases} \tag{17}$$

Thus, a set of coefficients can be calculated following the same principle of the correlation index as in (10) with:

$$C_n = \frac{2}{\pi} \int_{-1}^{1} \frac{f(x) T_n(x)}{\sqrt{1-x^2}} dx \qquad n = 1, 2, 3\ldots \tag{18}$$

And the synthesis of the orthogonal function can be better defined inside [−1, 1] through (Abramowitz & Stegun, 1965; Zhang & Jin, 1996; Lathi, 2005):

$$f(x) \approx (1/2)\, C_0 T_0(x) + C_1 T_1(x) + \ldots + C_n T_n(x) \tag{19}$$

B- Chebyshev polynonial of the second kind

The second kind of Chebyshev polynomials are a set of formulations similar to the first one above first statement given by (Abramowitz & Stegun, 1965; Zhang & Jin, 1996):

$$U_n(x) = \frac{\sin[(n+1)\cos^{-1} x]}{\sin(\cos^{-1} x)} \tag{20}$$

Where its direct numerical implementation can be obtained conveniently from the following recurrence relation, (Abramowitz & Stegun, 1965; Zhang & Jin, 1996):

$$U_0(x) = 1 \qquad U_1(x) = 2x$$

and:

$$U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x) \tag{21}$$

In addition, the Chebyshev polynomials as (21) are orthogonal on [−1, 1], with respect to the weight $w(x) = \sqrt{1-x^2}$, only if their inner product satisfies (18).

A similar set of coefficients can be calculated with (11) using the following (Abramowitz & Stegun, 1965):

$$C_n = \frac{2}{\pi}\int_{-1}^{1}\sqrt{1-x^2}\, f(x)U_n(x)dx \qquad n = 1, 2, 3, \ldots \tag{22}$$

The synthesis of the approximated function is given by the following orthogonal polynomial inside [−1, 1] which has been implemented here in MATLAB (MATLAB Version: 9.13.0.2049777 (R2022b), 2025) (Abramowitz & Stegun, 1965):

$$f(x) \approx C_0U_0(x) + C_1U_1(x) + \ldots + C_nU_n(x) \tag{23}$$

C- Legendre function approximation

The Legendre functions or polynomials are described conveniently through the Rodrigues's formula for an integer degree as, (Abramowitz & Stegun, 1965):

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n] \tag{24}$$

for all $x \in [-1, 1]$.

For the numerical implementation in (MATLAB Version: 9.13.0.2049777 (R2022b), 2025) is better to follow the recursive relation, (Abramowitz & Stegun, 1965):

$$P_{n+1}(x) = \frac{(2n+1)xP_n(x) - nP_{n-1}(x)}{n+1} \tag{25}$$

where:

$P_0(x) = 1$ and $P_1(x) = x$

By using (11), the vector coefficients for the series approximation are given by Abramowitz & Stegun (1965):

$$C_n = \frac{\int_{-1}^{1} f(x)P_n(x)dx}{\int_{-1}^{1} P_n^2(x)dx} = \frac{2n+1}{2}\int_{-1}^{1} f(x)P_n(x)dx \tag{26}$$

For $n = 1, 2, 3 \ldots$

And the Fourier-Legendre series solution defined inside the range [−1, 1] for an arbitrary continuous function $f(x)$ is given by Abramowitz & Stegun (1965):

$$f(x) \approx \sum_{n=0}^{\infty} C_nP_n(x) \tag{27}$$

Harmonic electrical signals orthogonal sets

Classical harmonic analysis with applications in electrical engineering has been performed for many years based on The Fourier Theory (Arrillaga, 2003; Fourier, 1822). In common electrical engineering practices, the assumption that the utility supply voltage can be treated as a pure harmonic sinusoidal signal considerably simplifies the system modelling problem, as solving it through a phasor equivalent model can result in a much simpler one. However, for a non-balanced electrical equivalent circuit model or a non-linear problem, the traditional single sinusoidal approach sometimes yields a numerically inefficient or inaccurate solution.

This article explores the properties and characteristics of the orthogonal decomposition of two types of electrical signals: one synthesized in MATLAB (harmonic and interharmonic) and the other a real voltage signal captured by direct measurement at the output of an uninterruptible power supply UPS using a Tektronix DPO 4104 digital oscilloscope.

In the case study of the signal type synthesized in MATLAB (one harmonic and one interharmonic control signal), uniformly distributed pseudo-random noise was added using the Mersenne Twister generator with seed 0 (MATLAB Version: 9.13.0.2049777 (R2022b), 2025).

1st Test harmonic signal (Control signal)

According to recommendations in the IEEE 519-1992 standard, a test harmonic noise signal is implemented numerically over 12 periods of time (200 milliseconds). The voltage-simulated electrical signal has a power frequency of f = 60 Hz and 754 time-samples, using a Nyquist sampling rate that is twice the highest frequency present in the signal ($f_s = 2\times5\omega_0$), with $\omega_0 = 2\pi f$. This implementation was conducted in MATLAB version 9.

Uribe previously documented a comparable test signal in 2024, which is quantitatively implemented in the following expression. This expression was determined to be the control signal because the frequencies are clearly evident in each sinusoidal signal argument, forming the fundamental, second, and third harmonics, respectively.

$$v(t) = \sin(\omega_0 \cdot t) + 0.50 * \sin(2\omega_0 \cdot t) + \ldots$$

$$\ldots + 0.30 * \sin(5\omega_0 \cdot t) + 0.1 * randn\,(size\,(t)) \tag{28}$$

The synthesized voltage signal in (28) and its estimated bar plot spectrum sequence, synthesized with 65 series terms according to the Fourier Theory (FFT, Lathi, 2005), are depicted below in Figures 2 and 3, respectively.

As illustrated in Figure 3, the synthesized voltage signal exhibits a high harmonic and pseudo-random noisy levels along a normalized harmonic axis (dimension-less) concerning the fundamental frequency of f= 60 Hz, which is analogous to the frequency provided by the government utility power supply voltage (MATLAB Version: 9, 2025).

The predicted magnitude of the harmonics aligns precisely with the values outlined in expression (28), a calculation that was executed in a computer processing time of 0.00526820 seconds (Uribe, 2024; Fourier, 1822).

The additive pseudo-random noise of signal (28) is estimated for verification of the accuracy in this paper with the signal-to-noise ratio (SNR), the total harmonic distortion (THD) and the signal to noise and distortion ratio (SINAD) of the signal defined as (Arrillaga, 2003; Bollen & Gu, 2006):

$$SNR = 10 \cdot \log_{10}\left(\frac{P_{fund}}{Var_{noise}}\right) \tag{29}$$

$$THD = 10 \cdot \log_{10}\left(\frac{P_{harm}}{P_{fund}}\right) \tag{30}$$

$$SINAD = 10 \cdot \log_{10}\left(\frac{P_{fund}}{P_{harm} + Var_{noise}}\right) \tag{31}$$

Where P_{fund} is the power of the fundamental component of $v(t)$ in (28), Pharm is the power of the harmonic components in (28) and Varnoise is the variance of the additive noise of 0.1 (Ribeiro *et al.*, 2014; Bollen & Gu, 2006).

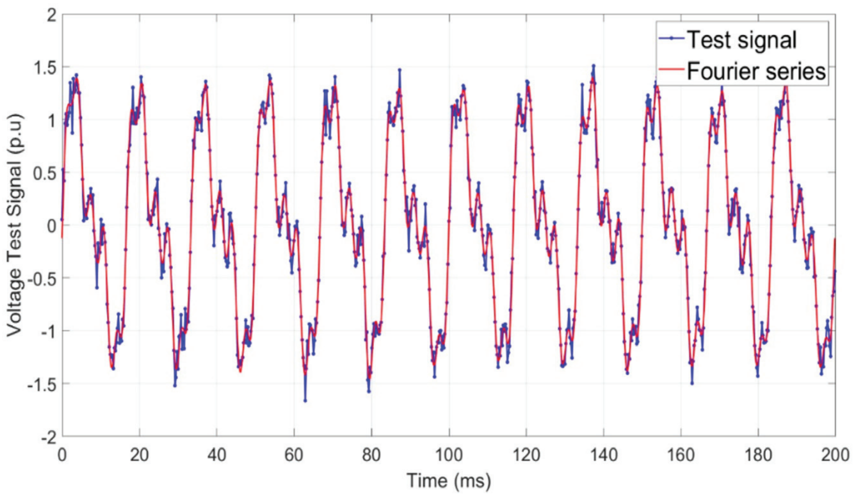

Figure 2. Fourier synthesis with 65 components of the test signal $v(t)$ in (28).

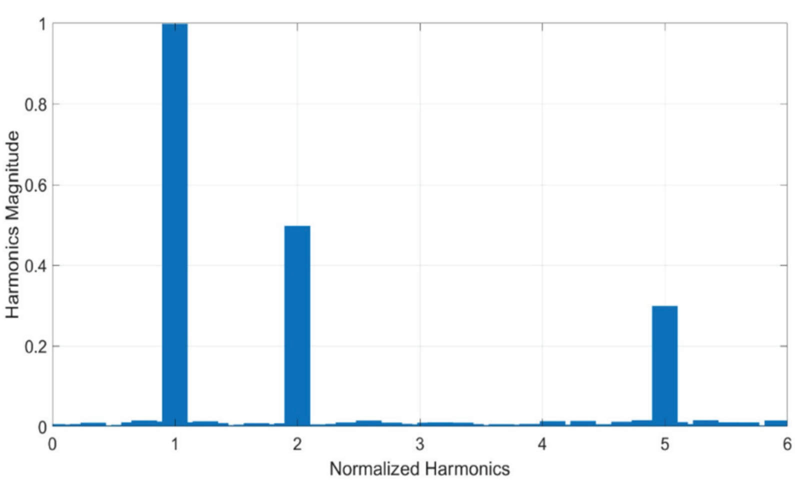

Figure 3. Normalized harmonic content for the test voltage $v(t)$ signal in (28) using Fourier series.

Table 1 specifies the analytical definitions for the signal distortion and noise parameters taken from the direct implementation of (29)-(31) of v(t) and also for comparison through the computing solution of MATLAB, from which one can see that both results are in good agreement.

The initial calculations were obtained using an HP Compaq 6200 Pro MT personal computer (PC) with Intel(R) Core (TM) i7-2600 CPU @ 3.40GHz RAM 8.0GB, Operative system 64bits, Windows 7 Professional, running MATLAB (MATLAB Version: 9.13.0.2049777 (R2022b), 2025).

Table 1. Comparison of the SNR, THD and SINAD of test base signal *v*(*t*) in (28).

	Analytical definition	MATLAB estimation
SNR	17.7135	16.9897
THD	− 4.5210	− 4.6852
SINAD	4.3651	4.4370

The Chebyshev polynomial of the first kind has been implemented as an evidence of the accuracy of the series approximation. Figure 4 shows the resulted synthesis using 200 polynomial terms of Chebyshev of the first kind using (18) and (19), while Figure 5 shows the magnitude of the harmonics bar plot sequence synthesized according to the Fourier Theory for the voltage signal approximated with Chebyshev series 1. The achieved accuracy is very high obtained in an elapsed computer processing time of 0.06536250s, only.

The Legendre series approximation in (26) and (27) has been implemented here with 187 terms for the harmonic test signal v(t) in (28), which results are shown in Figure 6 for the time domain approximation and in Figure 7 for the magnitude of the harmonic content according to the Fourier Theory for the voltage signal approximated with Legendre series.

The achieved accuracy of the Legendre series approximation is also very high obtained in an elapsed computer processing time of 0.05555610s.

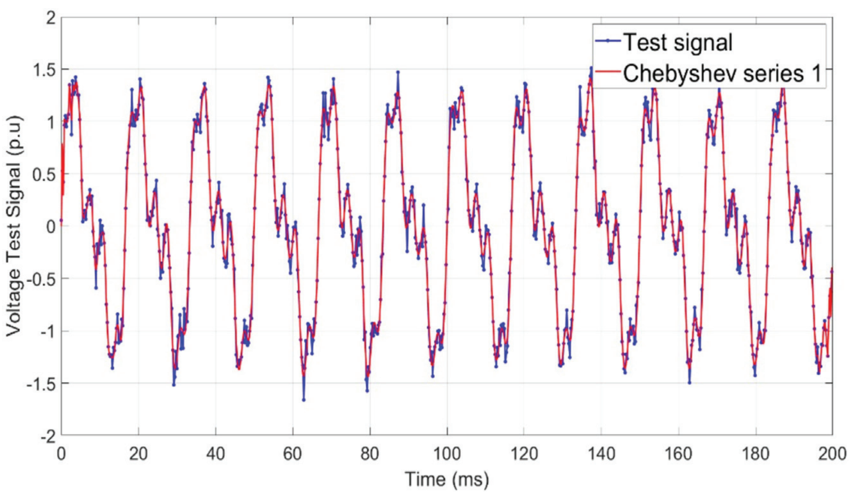

Figure 4. Chebyshev synthesis with 200 components of the test signal *v*(*t*) in (28).

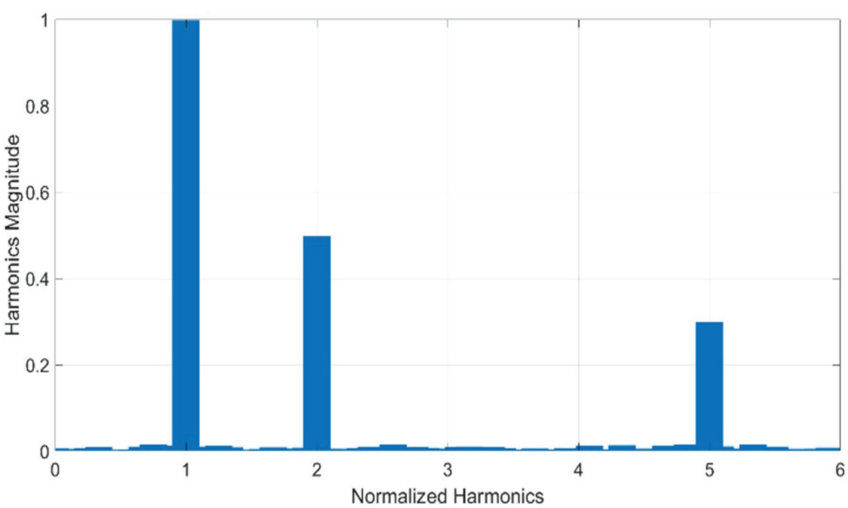

Figure 5. Normalized harmonic content for the test voltage *v*(*t*) signal in (28) using Chebyshev 1 series.

As can be seen here, in the study case of the harmonic signal in this paper section, the results obtained with the Chebyshev polynomials and the Lagrange series are very similar in magnitude and frequency to the harmonic content shown in Figure 5 and 7, with respect to the Fourier series solution in Figure 3 as expected, respectively.

The qualitative analysis as a function of harmonic content allowed us to know, the minimum number of terms needed to construct the harmonic spectrum of each approximation by observing the magnitudes of the harmonics corresponding to (28) as shown graphically in Figures 2, 4 and 6.

It should be noted that the test conditions in this section were performed in the presence of pseudo-random noise, in this case numerical, but in resemblance to the noise situation present in the utility power supply voltage signal.

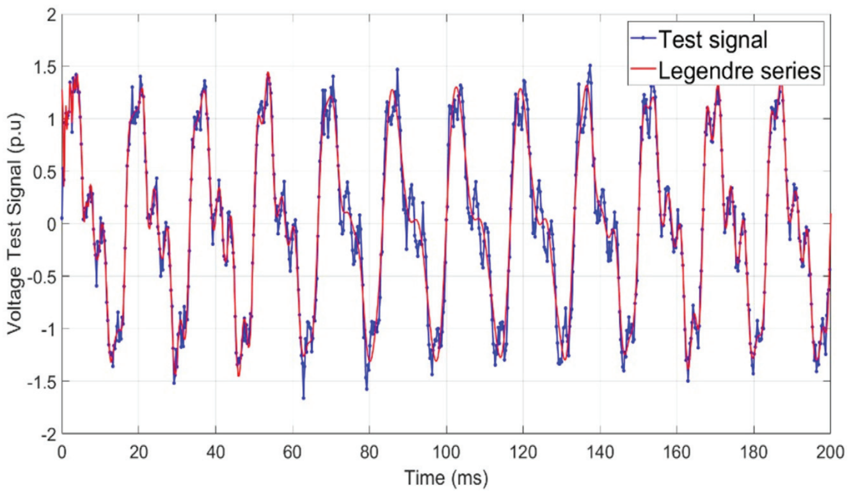

Figure 6. Legendre synthesis with 187 components of the test signal $v(t)$ in (28).

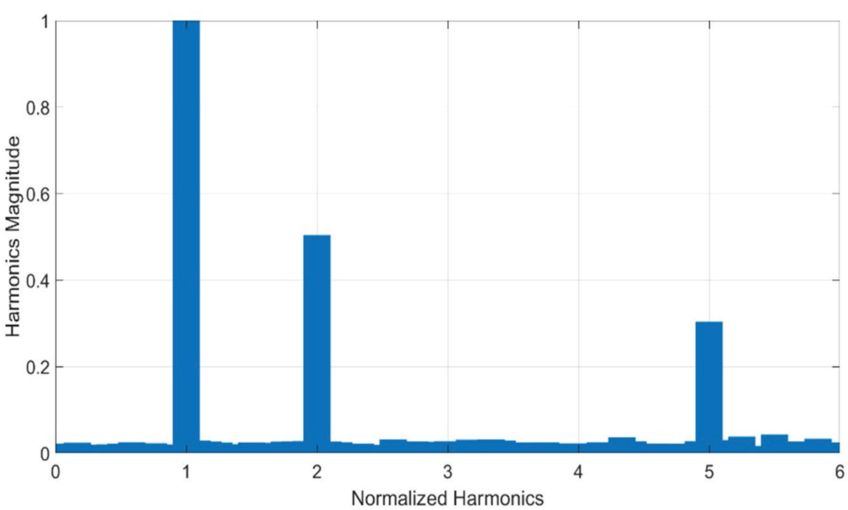

Figure 7. Normalized harmonic content for the test voltage $v(t)$ using 187 terms of Legendre series.

2nd Test inter-harmonic signal

In the second case of analysis, an interharmonic signal is synthesized numerically in MATLAB® according to indications in IEEE 519-1992, 1993. The signal is simulated over 12 periods of time (200 ms) and has a fundamental frequency of f = 60 Hz, with 1,508 time-samples. The Nyquist sampling rate of $fs = 2*10*\omega_0$, with $\omega_0 = 2\pi f$, is used to generate the signal. The following mathematical expression with the coefficients determined is used to represent the signal:

$$v(t) = \sin(\omega_0 \cdot t) + 0.50 * \sin(3\,\omega_0 \cdot t) + \ldots$$

$$\ldots + 0.30 * \sin(5.35\,\omega_0 \cdot t) + 0.1 * randn(size(t)) \quad (32)$$

As illustrated in Figure 8, the second test voltage signal $v(t)$ was implemented in accordance with equation (32). The Fourier series approximation is also derived from this signal, as illustrated in the same figure. This calculation utilizes 66 terms and requires a processing time of only 0.00752920 seconds. This approximation has been shown to be notably effective, even when confronted with a substantial level of pseudo-random noise from the high signal source. This effectiveness is illustrated in Table 2 for three different signal-to-noise and distortion parameter ratios: SNR, THD, and SINAD.

The magnitude of the harmonics according to the Fourier Theory of the fundamental and the third components of the voltage signal in (32) and the inter-harmonic in 5.5 of the normalized scale of frequency (f/60) are shown with high accuracy with the implemented Fourier series in Figure 9.

The Chebyshev polynomial approximation has been implemented for the inter-harmonic voltage v(t) signal in (32) using 220 terms in 0.29311260s of computer processing time.

Figure 10 shows the time domain accuracy of the polynomial Chebyshev series approximation for the 2nd voltage test signal in (32). In addition, the magnitude of the harmonics voltages $v(t)$ according to the Fourier Theory along a normalized frequency (f/60) is shown in Figure 11 for this 2nd test signal. Also, the presence of noise is evidenced in the bottom of the plot, showing values very close to zero of the frequency magnitude scale. This is known as a part of the frequency leakage dispersion. In this case, it doesn't represent an obstacle or a numerical problem for the harmonic identification process result.

Figure 12 shows the results for the Legendre function approximation has also implemented for this 2nd test case of inter-harmonic voltage v(t) signal with 220 components in 0.30170290s of computer processing time. The magnitude of the fundamental and third harmonics and of the inter-harmonic of 5.5 frequency components are evidenced in Figure 13 in bar plot along a normalized frequency (f/60) axis with high accuracy. It can be seen from Figures 12 and 13, that the accuracy of

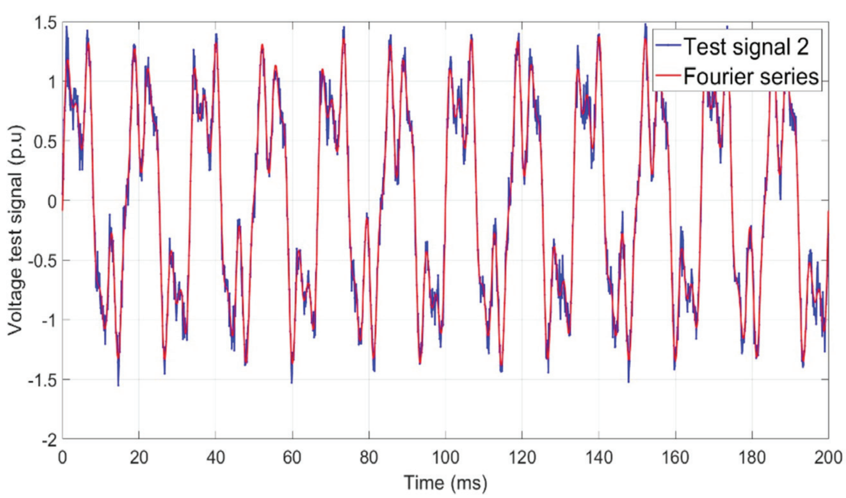

Figure 8. Fourier synthesis with 66 components of test signal $v(t)$ in (32).

Table 2. Comparison of the SNR, THD and SINAD of the second test base signal $v(t)$ in (32).

	Analytical definition	MATLAB estimation
SNR	9.7226	16.9897
THD	-6.1610	-4.6852
SINAD	4.5893	4.4370

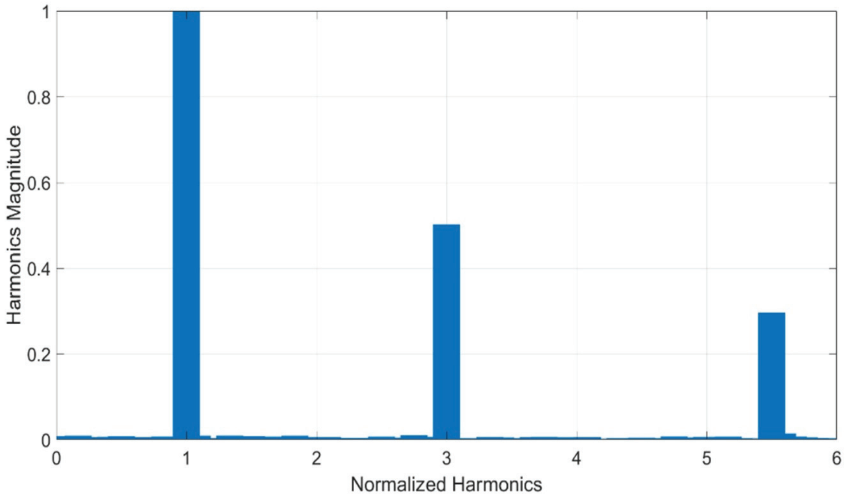

Figure 9. Normalized harmonic content for the test voltage $v(t)$ signal (32) using Fourier series.

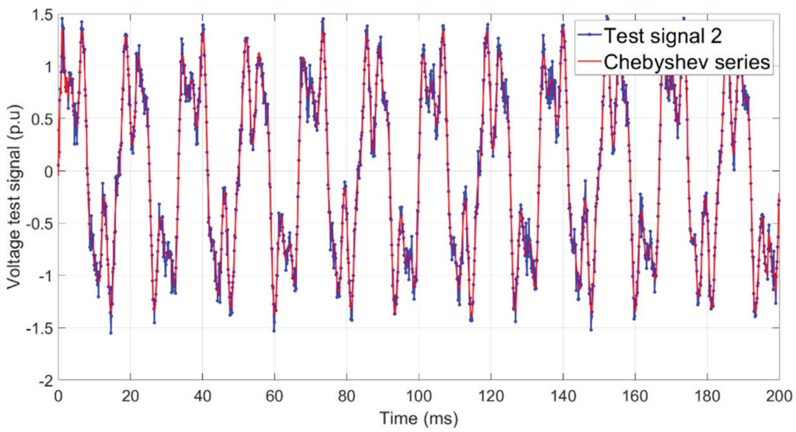

Figure 10. Chebyshev 1 synthesis with 220 components of test signal $v(t)$ in (32).

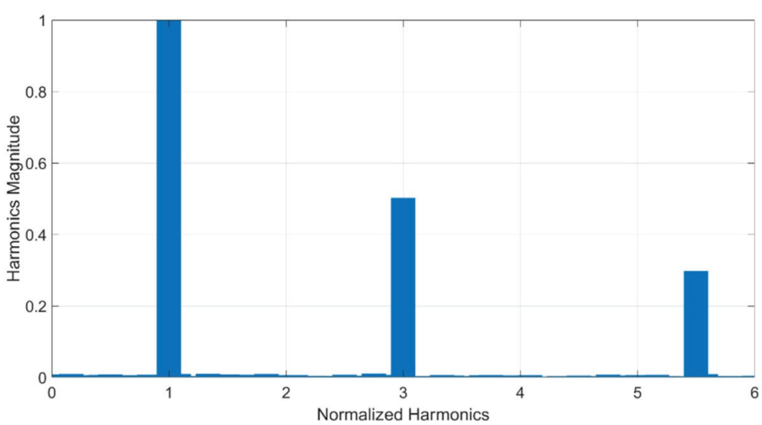

Figure 11. Normalized harmonic content for the test voltage $v(t)$ using Chebyshev 1 series.

Legendre series approximation in (26) and (27) applied to the identification of inter-harmonic electrical signals is high for electrical signal applications.

For the case of analyzing interharmonic electrical signal, it can be seen that the above approximations presented excellent results in a very short computer processing-time. However, in both time and harmonic type of plot for each one technique there is a kind of frequency leakage dispersion, due to the presence of pseudo-random noise in the voltage test signal $v(t)$. For harmonic analysis standard applications, this is not very significative for study of harmonics propagation

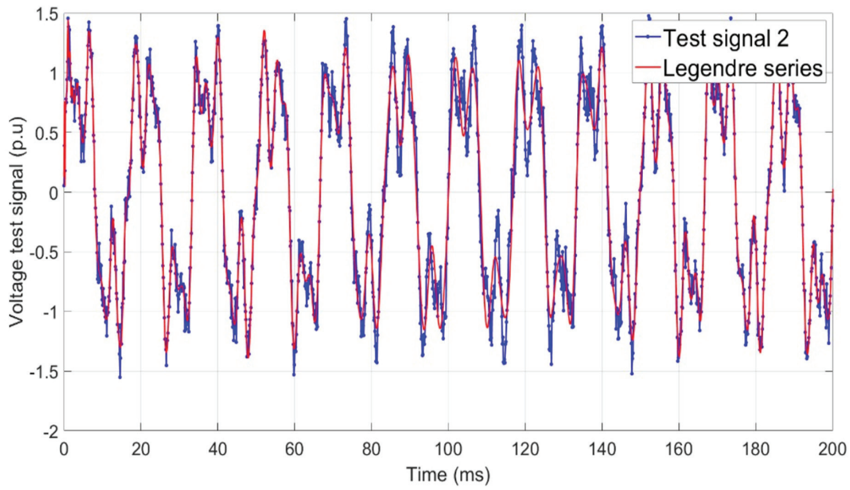

Figure 12. Legendre synthesis with 220 components of test signal *v*(*t*) in (32).

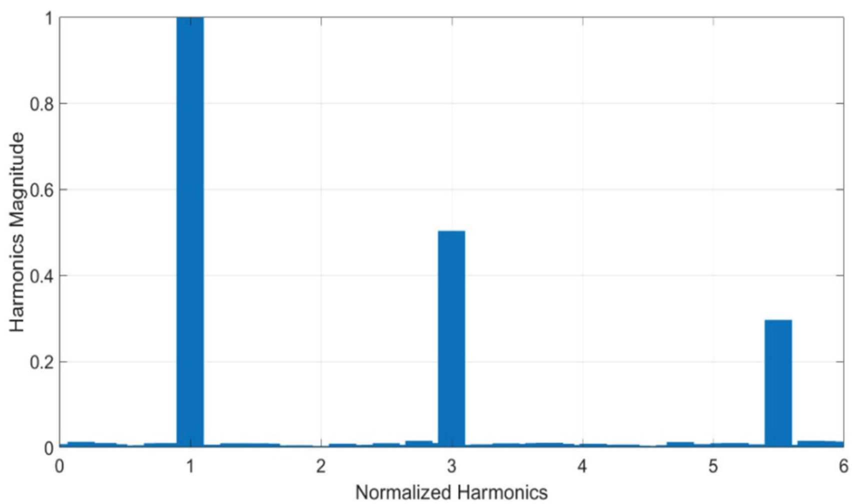

Figure 13. Normalized harmonic content for the test voltage *v*(*t*) using Legendre series.

or for electromagnetic induction problems (IEEE Recommended Practice and Requirements for Harmonic Control in Electric Power Systems, in IEEE Std 519-2014 (Revision of IEEE Std 519-1992), 2014).

3rd Test analysis of actual harmonic signal

A harmonic voltage test signal measurement has been obtained in our LAB using a DPO4104 Tektronix digital phosphor oscilloscope with a massive storage sampling rate of 5 GS/s on all of the 4 channels and 1 GHz of bandwidth, ideal for applications of investigation of the transient phenomena and spectral analysis as the one illustratively shown in Figure 14 (DPO 4104 Digital Phosphor, Oscilloscopes Tektronix 4000 Series Digital).

To test the here proposed harmonic analysis method we analyzed the voltage response of an uninterruptible power supply unit (UPS) shown in Figure 15 with 127 V voltage amplitude in 40 ms of duration recorded with 100 k samples using a frequency sampling of 25 kS/s.

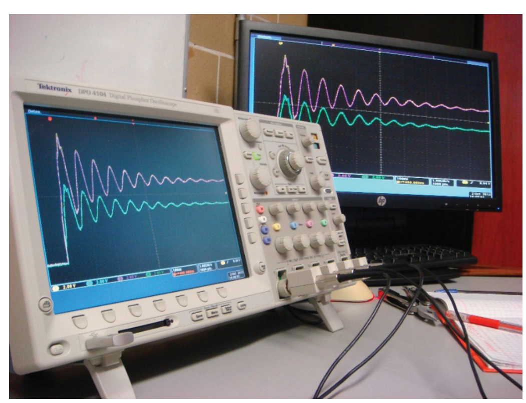

Figure 14. Digital phosphor oscilloscope (DPO 4104 Tektronix) for measuring high-speed electrical signals and massive storage as transients and harmonic signals.

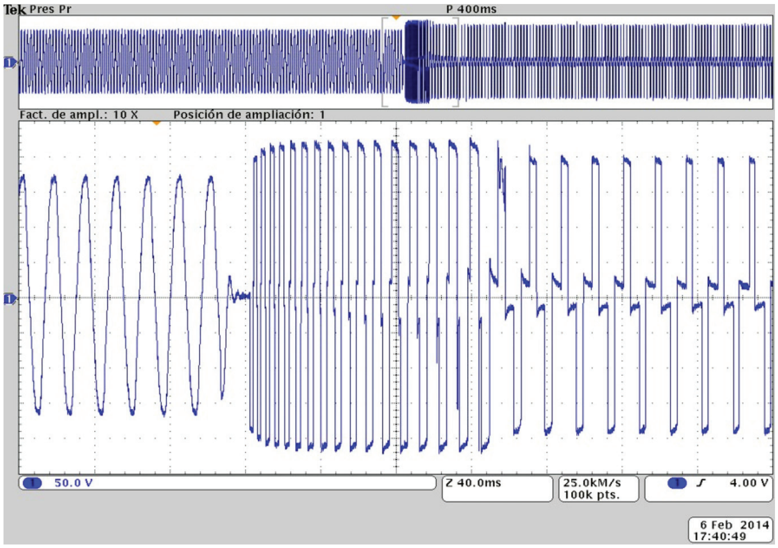

Figure 15. Voltage perturbation measurement of an uninterruptible.

Power Supply Unit Response (UPS) at a charge centre

To test the here applied methodology, the measured voltage signal in Figure 15 is synthesized with the Fourier, Chebyshev 1 and Legendre series techniques with high accuracy as can be seen in Figure 16 with pronounced Gibbs phenomenon in each of the discontinuities of the actual signal (Lathi, 2005). However, in this case of analysis this phenomenon does not represents a numerical problem.

However, a filtering operation as the convolution FIR filter of 3rd or 4th order can easily significative decrease these oscillations (Lathi, 2005).

Figure 17 shows the first 10th harmonic information data window according the Fourier Theory for the actual voltage signal shown in Figure 15. The magnitude of the profile of the complete spectrum can be seen in a solid red line, while the bar plot of the harmonics present in the actual signal are evidenced at the bottom of the plot. It can be mentioned here, that both set of information are in a good agreement each other as can be shown.

In appearance, in Figure 17 there is a 4-sample delay with respect to the normalized frequency of the voltage power supply of 60 Hz in the horizontal axis. It is possible that the frequency leakage effects occur when the inter-harmonics are predominant, over the simple harmonics range components, as a part of the multiple frequency contain of the square wave form of the signal (Lathi, 2005).

Thus, the frequency leakage turns the Fourier synthesis in a notable loss of information with respect to the actual signal. In the authors' experience the frequency leakage occurs around the main sampling frequency of the signal (IEEE Recommended Practice and Requirements for Harmonic Control in Electric Power Systems, in IEEE Std 519-2014 (Revision of IEEE Std 519-1992), 2014).

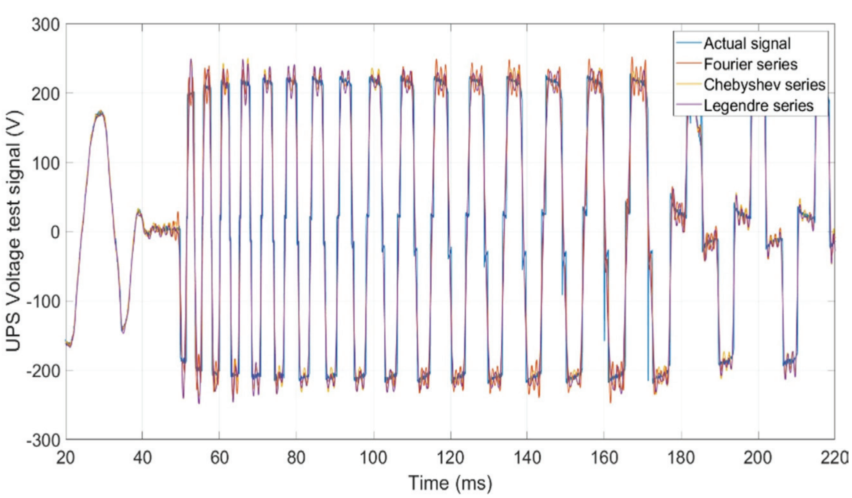

Figure 16. Synthesis of a UPS measurement voltage signal response.

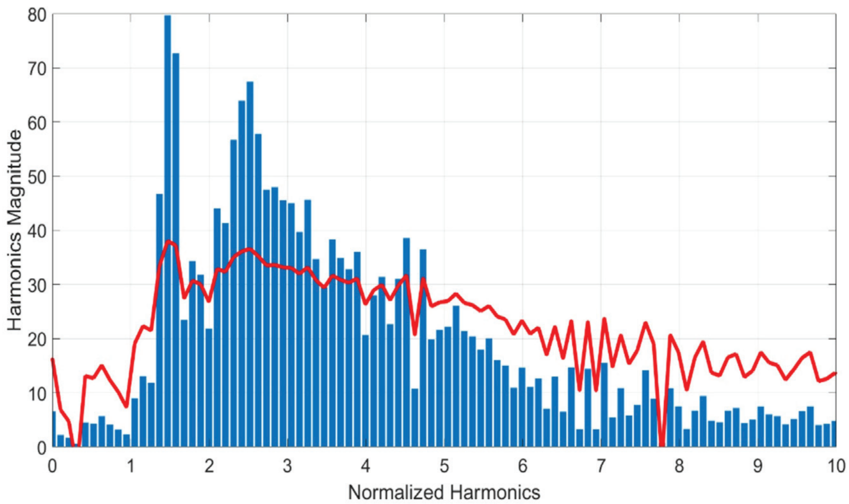

Figure 17. Normalized Fourier harmonic components (blue color) and distorted magnitude estimation (red color).

The behavior described in Figure 16 shows that the Generalized-Fourier orthogonal approximations of Chebyshev 1 and Legendre can achieve into a more accurate solution, Table 3 shows the synthesis parameter information for the case of treating with a UPS voltage signal containing harmonic and inter-harmonic components. However, more research has to be made in this regard to fully support this claim.

The dynamics of certain types of electrical signals obtained in cases by direct measurement can be analyzed through their time evolution by means of series of generalized Fourier, Chebyshev and Legendre functions with high accuracy and in a reduced computational processing time. The indices in the approximation in the sense of energy and correlation as shown in Table 3 confirm the accuracy on the obtained results in this work and can be extended to any other physical signal harmonic analysis.

As a power quality index, an abstraction of the estimated power spectrum of the voltage measurement signal in Figure 15 is shown in Figure 18 using the Fourier, Chebyshev, and Legendre approximations. The SNR factor estimates the relationship between the sinusoidal portion of the signal and the noise in decibels by calculating the ratio of its squared magnitude summed with that of the noise present in the signal. According to the obtained values of maximum difference of the SNR indices (±0.27 dB) and in the C_n correlation (14) of 0.0215, it is possible to verify that the technique developed here has a high precision in the application developed here for electrical signals.

Although the technique of function synthesis using generalized Fourier series approximations is a very promising idea due to the high levels of accuracy achieved in numerical signal processing implemented in MATLAB with a few harmonics, the same results are not achieved when studying real measurement signals. Most real signals obtained by measurement exhibit nonlinear and nonstationary behavior, eliminating any opportunity for obtaining a good representation using a Fourier series, even when using many harmonics in the series.

When studying electrical signals where the fundamental component (60 Hz) is known as a reference, the case can be analyzed in parts or sections of the signal. This technique is better known as the Fourier transform applied through short time windows, where the relationship between sampling time and frequency must comply with the Heisenberg uncertainty principle (Ribeiro *et al.,* 2014; Bollen & Gu, 2006).

Table 3. Actual signal synthesis parameters information**.**

	Signal energy	Correlation -C_n- (14)	CPU-time (s)	Series terms	Samples
Actual signal	6.20000912×103				5979
Fourier series	6.03822315×103	0.98691700	0.08766860	250	5979
Chebyshev series	5.80427432×103	0.96778333	4.29733480	250	5979
Legendre series	5.84760453×103	0.96542952	5.13245030	250	5979

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a)

b)

c)

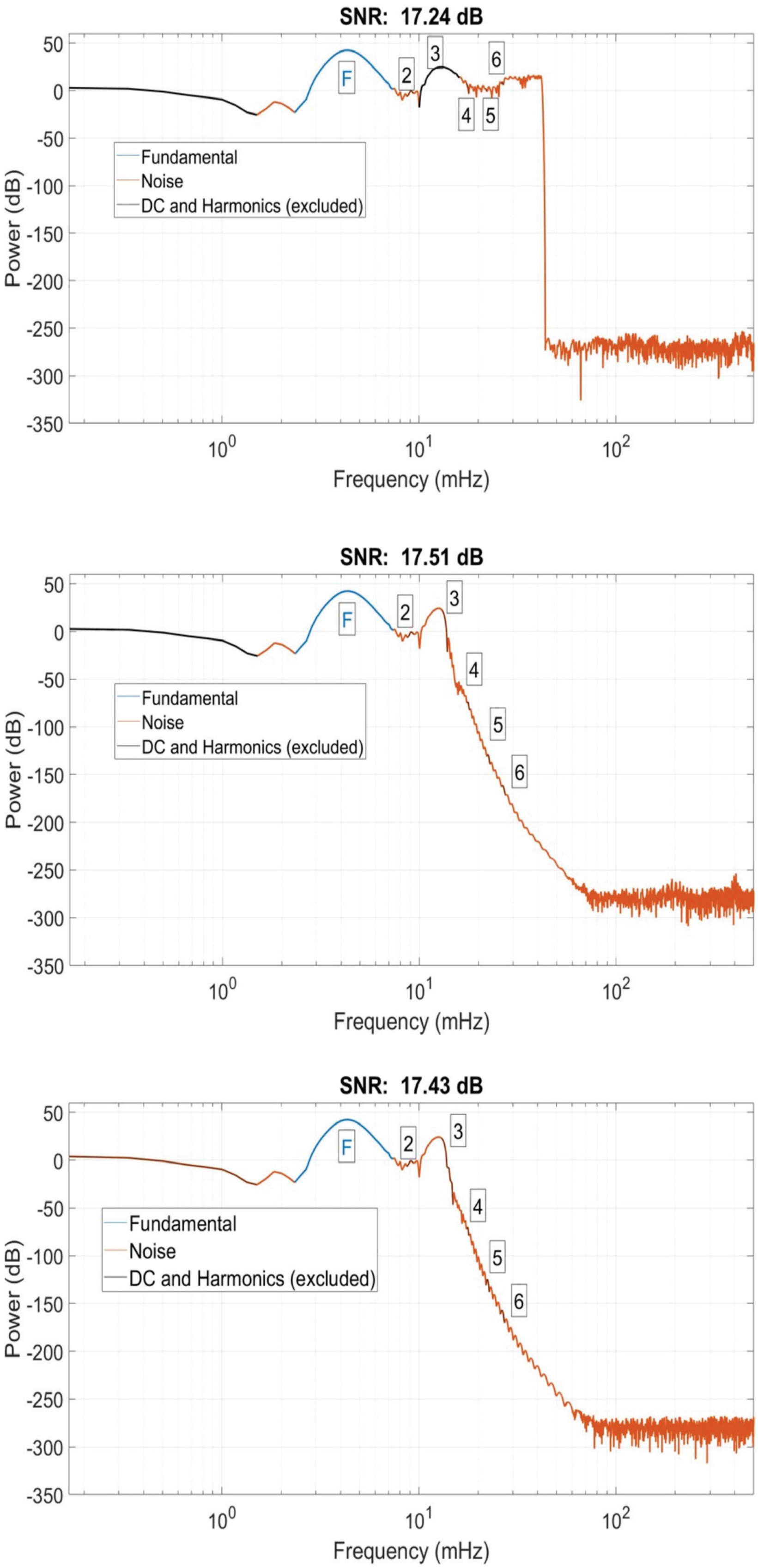

Figure 18. Power harmonic estimation and SNR for the first six components: a) Fourier approximant, b) Chebyshev approximant and c) Legendre series approximant.

Conclusions

In this paper a method to study dynamic electrical signals with noise by means of orthogonal basis vector decomposition was presented. By exploring the properties of the generalized Fourier, Chebyshev or Legendre series it is possible to synthesize with relatively few terms and in a short computational time accurate result. The approach presented here effectively exploits the properties of an orthogonal basis, even in the midst of noise, using a generalized Fourier series framework to improve understanding and accuracy. Central to this methodology is the principal component function, which enables the decomposition of noise and error functions based on both the correlation index and the approximate generalized Fourier series. We present compelling test applications analyzing harmonic, interharmonic, and real measurement signals using orthogonal Chebyshev polynomials, Legendre polynomial approximations, and Fourier series approximations. This method not only proves to be efficient with remarkable accuracy, but also maintains a low computational burden, which makes it very practical for real-world applications. By employing the correlation coefficient of polynomial and function approximations, we provide a qualitative comparison that highlights the accuracy of signal synthesis versus traditional Fourier theory.

In conclusion, the insights and features derived from this research allow us to extract crucial information about the propagation modal components, attenuation and velocities of the electrical signals under investigation. This advance is very promising for future applications in this field.

Acknowledgments

The author wishes to express his most sincere gratitude to The University of Guadalajara for being able to fulfill his dream of continuing to learn every day and to have the honor of teaching in the career of Electrical Mechanical Engineering in the engineering division of the University Center of Exact Sciences and Engineering (CUCEI) of the highest house of studies in western Mexico as the Alma Mater.

The author wishes to thank all those who have contributed different ideas and thoughts from the undergraduate engineering classrooms to the master of science in electrical engineering classes at CUCEI. Sort code 220203 and 330609.

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How to cite:

Uribe-Campos, F. A. (2025). Harmonics signal analysis with noise through orthogonal vector base decomposition method. *Ingeniería Investigación y Tecnología,* 26(04), 1-16. https://doi.org/10.22201/fi.25940732e.2025.26.4.029