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Robustness analysis of a PI-Posicast controller for second-order LTI systems with parameter uncertainty using Lyapunov-Krasovskii functionals

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Abstract

This paper presents a robust stability analysis of a Proportional-Integral Posicast (PI-Posicast) controller applied to a second-order underdamped system with parameter uncertainty. Unlike conventional frequency-domain methods, which provide only relaxed stability conditions, the proposed approach employs Lyapunov-Krasovskii functionals in the time domain to explicitly account for parameter uncertainty. This methodology enables the derivation of conservative robustness bounds that guarantee exponential stability even under perturbations in the plant parameters. The results demonstrate that the time-domain analysis offers stricter and more reliable robustness guarantees than frequency-domain techniques, ensuring system convergence within well-defined uncertainty limits. Numerical examples validate the theoretical findings and confirm the superior capability of time-domain analysis in handling uncertainty using the PI-Posicast controller.

Keywords: Posicast, stability, Lyapunov-Krasovskii, time-delay systems, time-domain analysis, parameter uncertainty.

Introduction

Time delays are inherent in many natural and engineered processes. They arise whenever signals, matter, energy, or information require a finite time to propagate, leading to phenomena such as instability, oscillations, and measurement noise. Examples span a wide range of domains, including social, technological, electronic, communication, biological, biomedical, robotic, and mechatronic systems, where delays may occur at the input, output, or both stages of transmission and reception. These effects can degrade overall system performance.

To address these challenges, control theory has developed dedicated tools for systems with delays. Classical studies focused on combustion processes, electrical networks, and communication channels (Bellman & Cooke, 1963; Cook, 1962, 1966). Subsequent research advanced the design of controllers that explicitly account for delays to preserve plant stability (Cooke & Grossman, 1982; Cooke & Turi, 1994; Fridman, 2014; Gun *et al.*, 2003; Gu *et al.*, 2005). Techniques such as accurate variable measurement and filtering have been central to mitigating disturbances and oscillations (Hung, 2003, 2007; Krasovskii, 1956).

Two complementary frameworks dominate the stability analysis of delayed systems: the frequency-domain approach and the time-domain approach (Bellman & Turi, 1994; La Salle & Lefschetz, 2012; Louisell, 1998; Neimark, 1949; Niculescu, 2001; Ramírez *et al*., 2015a, 2015b, 2017). Within the time-domain tradition, the Posicast filter has proven particularly effective for damping oscillations in underdamped second-order systems with delays. Pioneering studies demonstrated that when the damping coefficient is less than one, the Posicast compensator can suppress oscillatory phenomena despite the presence of infinitely many stable zeros (Razumikhin, 1956; Smith, 1957; Spiegel, 1965). Smith (1957) modeled a servomechanism to illustrate Posicast design and stability, while Spiegel (1965) extended the discussion by combining an integral controller with frequency-domain analysis. Although powerful, the classic Posicast strategy remains essentially open-loop, and subsequent proposals have explored its integration with PID control to enhance robustness.

In parallel, Lyapunov-based methods provide a broader theoretical foundation for stability analysis. The Lyapunov direct method addresses delay-free systems, whereas the Lyapunov-Krasovskii and Lyapunov-Razumikhin approaches incorporate explicit time delays. These methods yield conditions for asymptotic stability even under time-varying perturbations (Razumikhin, 1956; Vrančić & Oliveira, 2012), building on foundational results by Bellman & Cooke (1963) and Zubov (1961). More recent developments include complete-type Lyapunov-Krasovskii functionals tailored to proportional retarded systems (Villafuerte *et al.*, 2010, 2012; Kharitonov, 2012; Ojeda *et al.*, 2025).

Motivated by these advances, this work investigates the robustness of delayed systems within the Lyapunov-Razumikhin framework and proposes a PI-Posicast controller. By combining a Proportional-Integral feedback loop with a Posicast compensator, the controller stabilizes underdamped systems while enhancing robustness against constant disturbances. Using σ-stability criteria, the approach establishes bounds on unknown or time-varying perturbations under which the system remains asymptotically stable, providing a rigorous foundation for the design of controllers resilient to delays and uncertainties.

A PI-Posicast controller combines a Proportional-Integral (PI) feedback loop with a Posicast compensator, distinguishing it from traditional Posicast controllers. Its key feature is the ability to stabilize underdamped systems while enhancing robustness against constant disturbances, achieving zero steady-state error. Unlike conventional approaches, it is designed using modern σ-stability criteria, providing a rigorous framework for stability analysis and effective disturbance rejection.

Motivation for a time-domain robustness study: PI-Posicast control of second-order systems with unknown perturbations

In many engineering systems, the presence of time delays and uncertainties makes the design of robust controllers particularly challenging. Proportional-Integral (PI) Posicast control has proven effective for attenuating oscillations in second-order systems with delays; however, its performance is most often evaluated through frequency-domain analysis. While frequency-domain techniques are valuable for assessing stability and steady-state behavior, they provide only limited insight into the magnitude and dynamic influence of unmodeled parameters or uncertainties.

These limitations motivate the development of a time-domain methodology capable of describing the full transient evolution of the system and capturing the real impact of parameter uncertainty. A time-domain perspective also enables the determination of explicit robustness bounds, revealing the maximum disturbance level the system can tolerate without losing stability.

This work therefore presents a robustness study of a second-order system controlled under parameter uncertainty using a PI-Posicast strategy, within the Lyapunov-Razumikhin framework. By shifting the analysis from the frequency domain to the time domain, the proposed approach overcomes the limitations of classical frequency-based evaluations and provides explicit criteria for designing delay-tolerant controllers that remain stable under uncertain and time-varying perturbations.

In this study, the Proportional-Integral Posicast (PI-Posicast) controller, a type of delay-based control, is analyzed (Zaldívar & Ojeda, 2023). The controller is applied to a second-order linear time-invariant (LTI) system and consists of a Posicast compensator in cascade with a Proportional-Integral (PI) feedback loop. The system has been evaluated in the frequency domain using D-partition and σ-stability methods. A key advantage of this controller is its ability to maintain robustness when system parameters are subject to perturbations.

For the design of the robust PI-Posicast controller, a sub-damped second-order system is used, with parameters within a specified range $0 < \zeta < 1$, along with the Smith Posicast controller previously described. The goal is to validate that the PI-Posicast controller, in contrast to the standalone Posicast controller, offers improved robustness-specifically, the ability to mitigate the effects of constant external disturbances applied to the system. The PI-Posicast controller is defined as follows:

$$U(s) = \left[k_p + \frac{k_i}{s}\right]\left[\frac{1}{1+A} + \left(\frac{A}{1+A}\right)e^{-sh}\right] \tag{1}$$

Where:

k_p = proportional gain
k_i = integral gain
A = maximum overshoot given from response of the underdamped second-order system with natural period of the system's response to a step input and finally,
h = delay defined as a function of the natural period

The robustness of the controller is mathematically validated using the final value theorem (Zaldívar & Ojeda, 2023), which allows estimation of the steady-state output of the underdamped second-order system under the action of the PI-Posicast controller when subjected to external disturbances. This analysis involves determining whether the system can reject constant or slowly varying disturbances and return to a stable equilibrium.

Specifically, perturbations are introduced into the system, either as step disturbances or parameter uncertainties, and the behavior of the output signal is examined. By applying the final value theorem in the Laplace domain, it is possible to predict whether the system output converges to a finite value or diverges.

In addition, a frequency-domain approach is employed to assess how the controller handles disturbances across a range of frequencies. This includes analyzing the sensitivity function and the complementary sensitivity function, which provide insight into the system's ability to attenuate disturbances and maintain performance despite uncertainties.

The complete control scheme, including the interaction between the plant, the PI-Posicast controller, and the disturbance inputs, is illustrated in the block diagram in Figure 1. The diagram highlights the feedback and feedforward paths used to achieve robust performance.

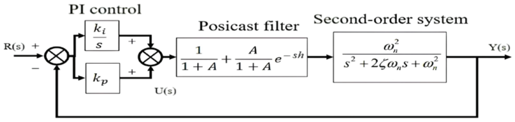

Figure 1. Block diagram for the Robust PI-Posicast controller.

The system employed is the second order in terms of the damping coefficient parameters determined in the range $0 < \zeta < 1$ and ω_n the natural frequency. The transfer function corresponding to the PI-Posicast control diagram shown in Figure 1 is given by:

$$G(s) = \left(\frac{k_i + k_p s}{s}\right)\left(k + (1-k)e^{-sh}\right)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

$$G(s) = \left(\frac{k_i + k_p s}{s}\right)\left(k + (1-k)e^{-sh}\right)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) =$$

$$\frac{\omega_n^2 kk_i + \omega_n^2 k_i(1-k)e^{-sh} + \omega_n^2 kk_p s + \omega_n^2 k_p(1-k)se^{-sh}}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

Obtaining the closed-loop transfer function $\frac{G(s)}{1+G(s)}$, ordering and grouping the terms, we have equation 2:

$$\frac{Y(s)}{R(s)} = \tag{2}$$

$$\frac{\omega_n{}^2 k_i\left(1-k\right)e^{-sh} + \omega_n{}^2 k_p\left(1-k\right)se^{-sh} + \omega_n{}^2 kk_p s + \omega_n{}^2 kk_i}{s^3 + 2\varsigma\omega_n s^2 + \left(\omega_n{}^2 + \omega_n{}^2 kk_p\right)s + \omega_n{}^2 kk_i + \omega_n{}^2 k_i\left(1-k\right)e^{-sh} + \omega_n{}^2 k_p\left(1-k\right)se^{-sh}}$$

Where $k = \frac{1}{1+A}$. To analyze the robustness of the second-order system with a PI-Posicast controller in the presence of parameter uncertainty in the plant, ζ and ω_n values of the proportional and integral gains of the external control loop are selected within the stability region.

Subsequently, a stability map is generated in the parameter space (ζ, ω_n) . This map allows for the identification of the range of values ζ and ω_n for the system parameters and control gains that ensure closed-loop stability under the given tuning. It is worth mentioning that these limits pertain to uncertain but constant parameters. The characteristic quasipolynomial of the closed-loop system (2) is:

$$p_{cl}(s,k_p,k_i) = s^3 + 2\varsigma\omega_n s^2 + \omega_n s + \omega_n\left(\left(1-k\right)e^{-sh} + k\right)\left(k_i + sk_p\right) \tag{3}$$

Using the D-partition method, the range of values (ζ, ω_n) for which the second-order plant with the PI-Posicast controller remains stable is determined.

The stability boundaries are described by the following equations. First, when $s = 0$ is substituted into the characteristic quasipolynomial (3) and equated to zero, the following expression is obtained:

$$p_{cl}(0,k_p,k_i) = \left(2k_i - 1\right)\omega_n{}^2 k_i = 0 \tag{4}$$

Solving the previous equation for ω_n gives:

$$\omega_n = 0 \tag{5}$$

and therefore, equation (5) defines the first stability boundary and delimits the crossing of a real root from a stable to an unstable zone.

On the other hand, when $s = j\omega$ substituting in equation (3) and separating the real part from the imaginary part and equating both parts to zero, we obtain:

$$\mathrm{Re}\left\{p_{cl}(j\omega,k_p,k_i)\right\} = \tag{6}$$
$$\omega_n\left(\omega_n k_i\left(\left(k-1\right)\cos\left(h\omega\right)+k\right)+\omega\left(\left(k-1\right)\omega_n k_p\sin\left(h\omega\right)-2\zeta\omega\right)\right)$$

$$\mathrm{Im}\left\{p_{cl}(j\omega,k_p,k_i)\right\} = \tag{7}$$
$$\omega_n^2\left(\omega k_p\left(\left(k-1\right)\cos\left(h\omega\right)+k\right)-(k-1)k_i\sin(h\omega)\right)-\omega^3+\omega_n^2\omega$$

Obtaining ζ from equation (6) implies:

$$\varsigma = \frac{\omega_n\left(k_i\left(\left(k-1\right)\cos(h\omega)+k\right)+\left(k-1\right)\omega k_p\sin\left(h\omega\right)\right)}{2\omega^2} \tag{8}$$

and ω_n from equation (7):

$$\omega_n = \pm\frac{\omega^{3/2}}{\sqrt{-\left(k-1\right)k_i\sin\left(h\omega\right)+\omega k_p\left(\left(k-1\right)\cos\left(h\omega\right)+k\right)+\omega}} \tag{9}$$

Equations (5), (8), and (9) describe the stability boundaries that divide the parametric space into stable and unstable regions. Figure 2 illustrates the stable parametric space, where the point (ω_n, ζ) and the point (1, 0.2) is selected. It is observed that the system is stable, as its roots are located in the left half of the complex plane.

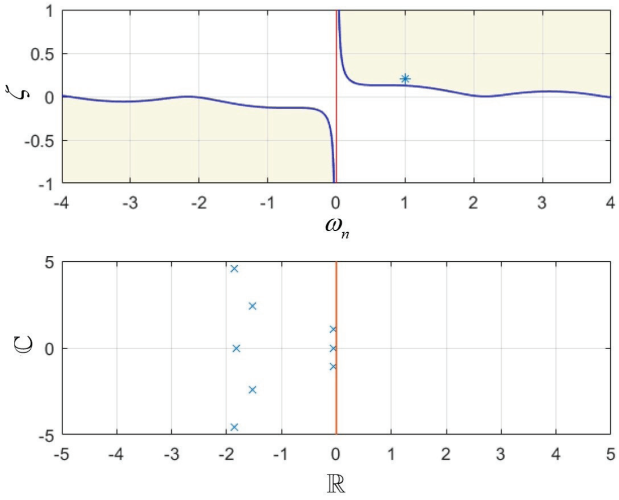

Figure 2. The stable zone of the parametric space (ω_n, ζ) using the parameters $k_p = 0.1810$, $k_i = 0.2742125$, $h = 3.2064$ and $k = 0.6550$, where x are the stable roots.

Figure 3 shows the system's output y (t). It can be observed that the output of the system with the PI-Posicast controller converges to the reference more slowly compared to the output of the system with the Posicast controller in closed loop. The main advantage of this tuning is that the transient response exhibits no oscillations.

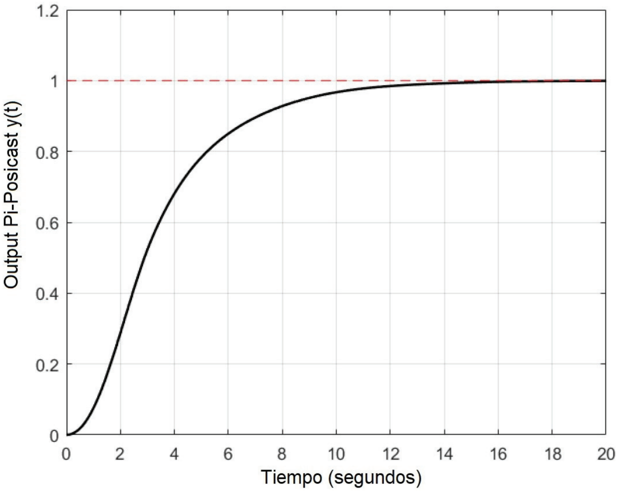

Figure 3. Output of system using PI-Posicast with $k_p = 0.1810$, $k_i = 0.2742125$, $h = 3.2064$, $k = 0.6550$, $\omega_n = 1$ and $\zeta = 0.2$.

Graphically, it can be observed that the values of ζ and ω_n can be disturbed in the ranges $0.1308 < \zeta < 1$ and $\omega_n > 0.2520$, while the closed-loop system maintains its stability.

Consider the following PI-Posicast controller parameters $h = 3.2064$, $k = 0.6924$, $k_p = 0.1810$, $k_i = 0.2742125$ and let the nominal system parameters (2) insert parameter uncertainty in order to study the effects of these perturbations on the output of the closed-loop system.

In Figure 4, it can be observed that when the system parameter (2) are inserted uncertainties, the output during the transient period exhibits an overshoot as the disturbance increases. However, the output eventually converges to reference due to the integral action of the PI-Posicast controller.

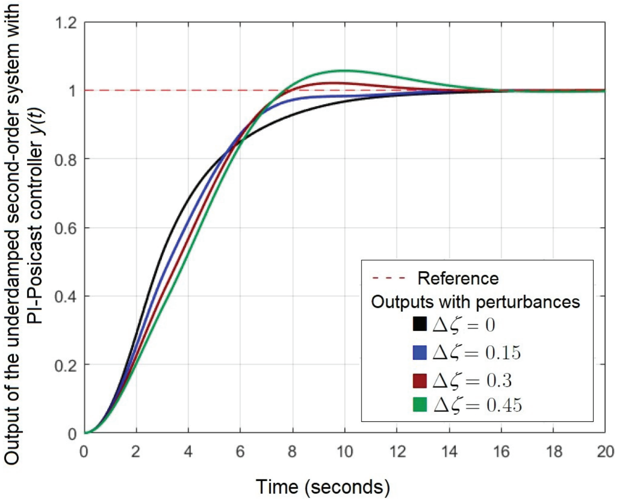

Figure 4. Output of the underdamped second-order system with PI-Posicast controller for $\Delta\zeta \in [-0.004, 0.004]$ and ω_n constant parameters.

Figure 5 shows that the output of system (2) converges to the reference value even in the presence of parameter uncertainty. However, during the transient period, oscillations are observed, which are not present in the output of the undisturbed system.

It is concluded that, when parameter uncertainties are present in the system with the PI-Posicast controller (Figure 1), the steady-state error remains unaffected and converges to zero. However, the presence of uncertainty introduces robustness limits that define a bounded region in the parameter space. Within these limits, the closed-loop underdamped second-order system retains stability, ensuring that the system states converge to their equilibrium points despite variations in the plant parameters. This confirms that the PI-Posicast controller preserves zero steady-state error while providing robustness against parameter uncertainty up to a defined threshold.

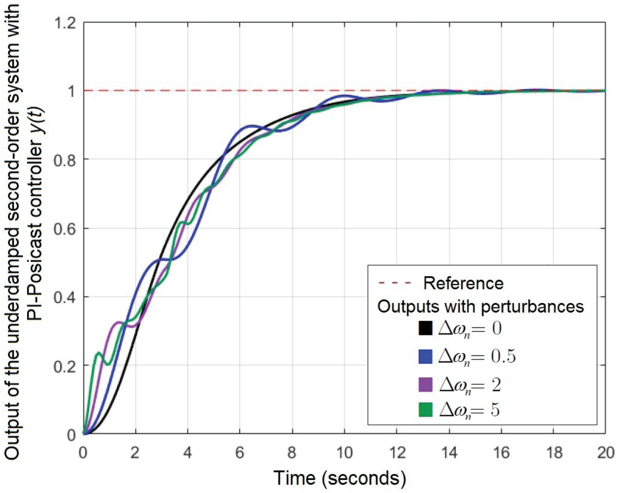

Figure 5. Output of the underdamped second-order system with PI-Posicast controller for ζ constant and $\Delta\omega_n \in [-0.0.021, 0.021]$.

The study of robustness in delayed feedback control systems is crucial for ensuring reliable performance under parameter uncertainty. According to Kharitonov (2012), frequency-domain methods provide less conservative stability conditions, allowing for larger allowable delays, but may not fully guarantee robustness against unknown perturbations. In contrast, time-domain approaches based on Lyapunov-Krasovskii functionals, while more conservative, ensure robust stability even in the presence of parameter uncertainty and varying delays.

Motivated by this trade-off, this work focuses on evaluating robustness levels for a closed-loop underdamped second-order system controlled by a PI-Posicast controller, using Lyapunov-Krasovskii functionals to obtain guaranteed stability bounds under parameter uncertainty. The choice of a Lyapunov-Krasovskii framework is justified by its ability to provide rigorous robustness guarantees, which is particularly important for systems with uncertain dynamics and delayed feedback.

In summary, the PI-Posicast controller exhibits robust performance by ensuring zero steady-state error even in the presence of parameter uncertainties. The robustness analysis, based on Lyapunov-Krasovskii functionals, establishes conservative yet reliable stability boundaries, defining the conditions under which the closed-loop underdamped second-order system converges to its equilibrium state despite parameter variations.

Robustness levels for a closed-loop underdamped second-order system by parameter uncertainty with a PI-Posicast controller obtained using Lyapunov-Krasovskii functionals

A robustness limit for a system with delay (Kharitonov, 2012) refers to the maximum level of parameter uncertainty that the system can tolerate without compromising stability. In this section, robustness levels are determined by analyzing disturbances in both the nominal and perturbed system parameters (Figure 1) in the time domain. This approach quantifies the extent of perturbation the system can withstand before the closed-loop system loses stability. Additionally, Kharitonov's method is particularly useful in the presence of parameter uncertainty, as it provides a framework for analyzing the system's behavior under worst-case scenarios, ensuring stability even in the presence of unmodeled disturbances or unforeseen parameter variations.

This study provides sufficient conditions that guarantee the closed-loop stability of the system under parametric perturbations. Therefore, the approach is inherently conservative and does not necessarily determine the true maximum tolerable disturbances. In other words, the time-domain analysis based on Lyapunov-Krasovskii functionals does not yield exact robustness limits; instead, it establishes sufficient conditions that ensure stability under parameter variations. Consequently, the results are conservative, guaranteeing stability even in worst-case scenarios but not necessarily capturing the system's true maximum robustness margins.

By evaluating the system's response to parameter uncertainty, the stability boundaries can be identified, providing insights into the resilience of the system under uncertain conditions. This method also highlights the critical parameters whose variations most significantly affect system stability, enabling targeted design improvements to enhance robustness. Ultimately, the objective is to determine the permissible range of disturbances that maintains system performance while ensuring that the system remains stable and converges to its equilibrium state. By applying the inverse Laplace transform to the characteristic equation (2), the following differential equation is obtained:

$$\begin{aligned}&\dddot{y}(t)+2\zeta\omega_n\ddot{y}(t)+\left(\omega_n+\omega_n^2k_1k_p\right)\dot{y}(t)+\omega_n^2k_1k_iy(t)+\\&\qquad\omega_n^2k_i(1-k_1)y(t-h)+\omega_n^2k_p(1-k_1)\dot{y}(t-h)\\&\quad=\omega_n^2k_1k_p\dot{r}(t)+\omega_n^2k_1k_ir(t)+\omega_n^2k_i(1-k_1)r(t-h)+\omega_n^2k_p(1-k_1)\dot{r}(t-h)\end{aligned}\tag{10}$$

For the regulation problem, a constant reference $r(t) = r$, $t \geq -h$ is considered, where both the instantaneous and delayed derivatives of the constant reference are zero. Consequently, the delayed values of the reference are equal to *r*, and the equation above reduces to:

$$\begin{aligned}&\dddot{y}(t)+2\zeta\omega_n\ddot{y}(t)+\left(\omega_n+\omega_n^2k_1k_p\right)\dot{y}(t)+\omega_n^2k_1k_iy(t)+\\&\omega_n^2k_i(1-k_1)y(t-h)+\omega_n^2k_p(1-k_1)\dot{y}(t-h)=\omega_n^2k_1k_pr+\\&\omega_n^2k_i(1-k_1)r.\end{aligned}\tag{11}$$

The error is defined as:

$$\bar{e}(t)=r-y(t)\tag{12}$$

This implies that:

$$y(t)=r-\bar{e}(t)\tag{13}$$

$$\dot{y}(t)=-\dot{\bar{e}}(t)\tag{14}$$

$$\ddot{y}(t)=-\ddot{\bar{e}}(t)\tag{15}$$

$$\dddot{y}(t)=-\dddot{\bar{e}}(t)\tag{16}$$

Substituting equations (13), (14), (15), and (16) into the differential equation (10), we obtain the differential equation (17) of the closed-loop system associated with the error:

$$\begin{aligned}&\dddot{\bar{e}}(t)+2\zeta\omega_n\ddot{\bar{e}}(t)+\left(\omega_n+\omega_n^2k_1k_p\right)\dot{\bar{e}}(t)+\omega_n^2k_1k_i\bar{e}(t)+\omega_n^2k_i\\&\qquad(1-k_1)\bar{e}(t-h)+\omega_n^2k_p(1-k_1)\dot{\bar{e}}(t-h)=0\end{aligned}\tag{17}$$

Defining the states $x(t)=\left[\bar{e}(t)\quad\dot{\bar{e}}(t)\quad\ddot{\bar{e}}(t)\right]^T$, it has the following class of systems with time-varying perturbations:

$$\begin{aligned}&\dot{x}(t)=A_0x(t)+A_1x(t-h),\qquad t\geq 0\\&x(\theta)=\varphi(\theta),\qquad \theta\in[-h,0]\end{aligned}\tag{18}$$

Where the matrices A_0, A_1 are:

$$A_0=\begin{bmatrix}0&1&0\\0&0&1\\-\omega_n^2k_1k_i&-\omega_n^2(1+k_1k_p)&-2\zeta\omega_n\end{bmatrix}\tag{19}$$

$$A_1=\begin{bmatrix}0&0&0\\0&0&0\\-\omega_nk_i(1-k_1)&-\omega_n^2k_p(1-k_1)&0\end{bmatrix}\tag{20}$$

To calculate the equilibrium point $x^*(t) = [x_1^*(t)\ x_2^*(t)\ x_2^*(t)]^T$ of the state representation of the error equation, it is observed that for systems with delays, at equilibrium, the delayed and instantaneous variables reach the same value $x^*(t)$, and the derivative is zero. The equilibrium point is presented in the following system:

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-\omega_n^2k_1k_i&-\omega_n^2(1+k_1k_p)&-2\zeta\omega_n\end{bmatrix}\begin{bmatrix}x_1^*(t)\\x_2^*(t)\\x_3^*(t)\end{bmatrix} \tag{21}$$

$$+\begin{bmatrix}0&0&0\\0&0&0\\-\omega_n^2k_i(1-k_1)&-\omega_n^2k_p(1-k_1)&0\end{bmatrix}\begin{bmatrix}x_1^*(t)\\x_2^*(t)\\x_3^*(t)\end{bmatrix}$$

So:

$$x_2^*(t)=0 \tag{22}$$

$$x_3^*(t)=0 \tag{23}$$

$$-\omega_n^2k_ix_1^*(t)-\omega_n^2(1+k_p)x_2^*(t)-2\zeta\omega_nx_3^*(t)=0 \tag{24}$$

Then, the only equilibrium points of the closed-loop system associated with the error given by:

$$\begin{bmatrix}x_1^*(t)\\x_2^*(t)\\x_3^*(t)\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{25}$$

It is considered that the closed-loop system described here presents perturbations to the parameters ζ, ω_n, respectively. Given $\Delta\zeta, \Delta\,\omega_n \in \mathbb{R}$ such unknown perturbations; then the matrices A_0 and A_1 can be rewritten in terms of their perturbations as follow:

$$\overline{A}_0 =$$

$$\begin{bmatrix}0&1&0\\0&0&1\\-\left(\omega_n+(\Delta\omega_n)\right)^2k_1k_i&-\left(\omega_n+(\Delta\omega_n)\right)^2(1+k_1k_p)&-2\left(\zeta+(\Delta\zeta)\right)\left(\omega_n+(\Delta\omega_n)\right)\end{bmatrix}$$

$$\overline{A}_0 = A_0+\Delta_0 \tag{26}$$

Where:

$$\Delta_0 =$$

$$\begin{bmatrix}0&0&0\\0&0&0\\-k_1k_i\left[\left(2\omega_n(\Delta\omega_n)+(\Delta\omega_n)\right)^2\right]&-(1+k_1k_p)\left(2\omega_n(\Delta\omega_n)+(\Delta\omega_n)\right)^2&-2\left[\zeta(\Delta\omega_n)+\omega_n(\Delta\zeta)\right]+\left(\Delta\omega_n+(\Delta\omega_n)\right)\end{bmatrix} \tag{27}$$

And:

$$\overline{A}_1=\begin{bmatrix}0&0&0\\0&0&0\\-\left(\omega_n+(\Delta\omega_n)\right)^2k_i(1-k_1)&-\left(\omega_n+(\Delta\omega_n)\right)^2k_p(1-k_1)&0\end{bmatrix} \tag{28}$$

$$\overline{A}_1 = A_1+\Delta_1$$

With:

$$\Delta_1=\begin{bmatrix}0&0&0\\0&0&0\\-\left(2\omega_n(\Delta\omega_n)+(\Delta\omega_n)^2\right)k_i(1-k_1)&-\left(2\omega_n(\Delta\omega_n)+(\Delta\omega_n)^2\right)k_p(1-k_1)&0\end{bmatrix} \tag{29}$$

The terms Δ_0 and Δ_1 represent unknown matrices containing the parameter uncertainty. In summary, the perturbed system with PI-Posicast control loop is represented as follows:

$$\begin{aligned}\dot{x}(t)&=(A_0+\Delta_0)x(t)+(A_1+\Delta_1)x(t-h)\\ \dot{x}(t)&=(\overline{A}_0)x(t)+(\overline{A}_1)x(t-h)\end{aligned} \tag{30}$$

Where the equation (30) contains $\Delta\zeta, \Delta\omega_n \in \mathbb{R}$, such uncertainties with:

$$\|\Delta_0\| \le \rho_0 \tag{31}$$

$$\|\Delta_1\| \le \rho_1 \tag{32}$$

$\|\Delta_0\|$ and $\|\Delta_1\|$ are the induced norms of the matrix with uncertainties Δ_0 and Δ_1, and ρ_0 and $\rho_1 \in \mathbb{R}^+$.

The following lines detail the stability test in the time domain using the full-type functional with the closed-loop system (30). First, a corollary is presented that establishes a condition on the derivative of the full-type functional along the trajectories of the perturbed system (30). Then, analytical expressions are determined for the robustness bounds ρ_0 and ρ_1 of the uncertainty matrices (27) and (29).

Corollary (Kharitonov, 2012). The derivative of the full type functional along the trajectories of the system with uncertainty parameters (30) is of the form:

$$\frac{d}{dt}v_c(t) = -x^T(t)W_0x(t) - x^T(t.-h)W_1x(t-h)$$

$$-\int_{t-h}^{t} x^T(\theta')W_2x(\theta')d\theta' + 2\left[\Delta_0x(t)+\Delta_1x(t-h)\right]^T L(x_t)$$

Where:

$$L(x_t) = U(0)x(t) + \int_{t-h}^{t} U(-h-\theta' + t)A_1x(\theta')d\theta'$$

Proof. It defines each term of the full-time functional as follows:

$$v_c(x_t) = G_1(x_t) + G_2(x_t) + G_3(x_t) + G_4(x_t)$$

Where:

$$G_1(x_t) = x^T(t)U(0)x(t)$$

$$G_2(x_t) = 2x^T(t)\int_{-h}^{0} U(-h-\theta)A_1x(t+\theta)d\theta$$

$$G_3(x_t) = \int_{-h}^{0} x^T(t+\theta_1)A_1^T\left[\int_{-h}^{0} U\left(\theta_1-\theta_2\right)A_1x(t+\theta_2)d\theta_2\right]d\theta_1$$

$$G_4(x_t) = \int_{-h}^{0} x^T(t+\theta_1)\left[W_1+(h+\theta)W_2\right]x(t+\theta)d\theta$$

A lemma from Kharitonov (2012) is used, where it is observed that the derivative of the full-type functional along the trajectories of the perturbed system (25) is given by:

$$\frac{d}{dt}G_1(x_t) = x^T(t)\left[U(0)A_0 + A_0^TU(0)\right]x(t) + 2x^T(t)U(0)A_1x(t-h)$$

$$+2x^T(t)U(0)\Delta_0x(t) + 2x^T(t)U(0)\Delta_1x(t-h)$$

$$\frac{d}{dt}G_2(x_t) = 2x^T(t)U(-h)A_1x(t) - 2x^T(t)U(0)A_1x(t-h)$$

$$+2x^T(t)\int_{t-h}^{t} U'(t-h-\theta')A_1x(\theta')d\theta'$$

$$+2x^T(t)A_0^T\int_{t-h}^{t} U(-h-\theta'+t)A_1x(\theta')d\theta' + 2x^T(t-h).$$

$$A_1^T\int_{t-h}^{t} U(t-h-\theta')A_1x(\theta')d\theta'$$

$$+2x^T(t)\Delta_0^T\int_{t-h}^{t} U(-h-\theta'+t)A_1x(\theta')d\theta' + 2x^T(t-h)\Delta_1^T\int_{t-h}^{t} U$$

$$(t-h-\theta')A_1x(\theta')d\theta'$$

$$\frac{d}{dt}G_3(x_t) = 2x^T(t)A_1^T\int_{t-h}^{t} U(t-\theta')A_1x(\theta')d\theta' - 2x^T(t-h)A_1^T\int_{t-h}^{t} U$$

$$(t-h-\theta')A_1x(\theta')d\theta'$$

$$\frac{d}{dt}G_4(x_t) = x^T(t)\left[W_1+hW_2\right]x(t) - x^T(t-h)W_1x(t-h) - \int_{t-h}^{t} x^T$$

$$(\theta')W_2x(\theta')d\theta'$$

Ordering like terms:

$$\dot{v}_c(x_t) = x^T(t)\Big[U(0)A_0 + A_0^TU(0) + U(-h)A_1 + A_1^TU(h) + W_1$$

$$+hW_2\Big]x(t) + \Big\{2x^T(t)U(0)A_1x(t-h) - 2x^T(t)U(0)$$

$$A_1x(t-h)\Big\}$$

$$+\left\{2x^T(t-h)A_1^T\int_{t-h}^{t} U(t-h-\theta')A_1x(\theta')d\theta' - 2x^T(t-h)A_1^T\int_{t-h}^{t} U\right.$$

$$\left.(t-h-\theta')A_1x(\theta')d\theta'\right\}$$

$$+2x^T(t)\int_{t-h}^{t}\left[U'(t-h-\theta') + U(t-h-\theta') + U(t-\theta')\right]A_1x(\theta')d\theta'$$

$$-x^T(t-h)W_1x(t-h) - \int_{t-h}^{t} x^T(\theta')W_2x(\theta')d\theta'$$

$$+2\left[\Delta_0x(t)+\Delta_1x(t-h)\right]^T\left[U(0)x(t) + \int_{t-h}^{t} U(-h-\theta'+t)A_1x(\theta')d\theta'\right]$$

Using the symmetry property and algebraic properties:

$$U(0)A_0 + A_0^TU(0) - U(-h)A_1 + A_1^TU(h) = -W$$

$$W = W_0 + W_1 + hW_2$$

$$U'(t-h-\theta') = -A_0^T(t-h-\theta') - A_1^T(t-\theta')$$

And the theorem from Kharitonov (2012), the derivative along the trajectories of the perturbed system is given by:

$$\frac{d}{dt}v_c(x_t) = -x^T(t)W_0x(t) - x^T(t-h)W_1x(t-h) - \int_{t-h}^{t} x^T(\theta')W_2x(\theta')d\theta'$$

$$+2\left[\Delta_0x(t)+\Delta_1x(t-h)\right]^T\left[U(0)x(t)+\int_{t-h}^{t}U(-h-\theta'+t)A_1x(\theta')d\theta'\right] \quad (33)$$

The following theorem (Kharitonov, 2012) focuses on determining analytical expressions that establish conditions for the robustness bounds of the uncertainty matrices (27) and (29), using the result obtained in the corollary. It is defined as follows:

$$\bar{v} = \max_{\theta\in[0,h]}\|U(\theta)\|, \qquad a = \|A_1\|$$

Theorem (Kharitonov, 2012). Let the system be exponentially stable. Given the positive definite matrices W_1, W_2, and W_0, then the perturbed system (30) remains exponentially stable for every parameter uncertainty by matrix Δ_0, Δ_1 and $\|\Delta_0\| < \rho_0$, $\|\Delta_1\| < \rho_1$ if the following conditions are satisfied:

$$\lambda_{\min}(W_0) \geq 2\rho_0\bar{v} + \rho_0 ah\bar{v} + \rho_1\bar{v} \quad (34)$$

$$\lambda_{\min}(W_1) \geq \rho_1\bar{v} + \rho_1 ah\bar{v} \quad (35)$$

$$\lambda_{\min}(W_2) \geq \rho_0\bar{v}a + \rho_1\bar{v}a \quad (36)$$

Proof. By obtaining the upper bounds using the corollary and applying the lemma (Kharitonov, 2012), it follows that:

$$\left\|x^T(t)W_0x(t)\right\| \leq \lambda_{\min}(W_0)\|x(t)\|^2$$

$$\left\|x^T(t-h)W_1x(t-h)\right\| \leq \lambda_{\min}(W_1)\|x(t-h)\|^2$$

$$\left\|\int_{t-h}^{t} x^T(\theta')W_2x(\theta')d\theta'\right\| \leq \lambda_{\min}(W_2)\int_{t-h}^{t}\|x(\theta')\|^2 d\theta'$$

$$2\left\|x^T(t)\Delta_0^T U(0)x(t)\right\| \leq 2\rho_0\bar{v}\|x(t)\|^2$$

$$2\left\|x^T(t)\Delta_0^T\int_{t-h}^{t}U(-h-\theta'+t)A_1x(\theta')d\theta'\right\| \leq a\rho_0\bar{v}$$

$$\left(h\|x(t)\|^2 + \int_{t-h}^{t}\|x(\theta')\|^2 d\theta'\right)$$

$$2\left\|x^T(t-h)\Delta_1^T\int_{t-h}^{t}U(0)x(t)\right\| \leq \rho_1\bar{v}\left(\left\|x^T(t-h)\right\|^2 + \|x(t)\|^2\right)$$

$$2\left\|x^T(t-h)\Delta_1^T\int_{t-h}^{t}U(-h-\theta'+t)A_1x(\theta')d\theta'\right\| \leq a\rho_1\bar{v}$$

□

$$\left(h\|x(t-h)\|^2 + \int_{t-h}^{t}\|x(\theta')\|^2 d\theta'\right)$$

Now:

$$\left\|\dot{v}_c(x_t)\right\| \leq \left(-\lambda_{\min}(W_0) + 2\rho_0\bar{v} + ah\rho_0\bar{v} + \rho_1\bar{v}\right)\|x(t)\|^2$$

$$+\left(-\lambda_{\min}(W_1) + \rho_1\bar{v} + ah\rho_1\bar{v}\right)\|y(t-h)\|^2$$

$$+\left(-\lambda_{\min}(W_2) + a\rho_0\bar{v} + a\rho_1\bar{v}\right)\int_{t-h}^{t}\|x(\theta')\|^2 d\theta' \leq 0$$

And for the derivative to remain negative, the following condition must hold:

$$-\lambda_{\min}(W_0) + 2\rho_0\bar{v} + ah\rho_0\bar{v} + \rho_1\bar{v} \leq 0$$

$$-\lambda_{\min}(W_1) + \rho_1\bar{v} + ah\rho_1\bar{v} \leq 0$$

$$-\lambda_{\min}(W_2) + a\rho_0\bar{v} + a\rho_1\bar{v} \leq 0$$

Which implies:

$$\lambda_{\min}(W_0) \geq 2\rho_0\bar{v} + ah\rho_0\bar{v} + \rho_1\bar{v}$$

$$\lambda_{\min}(W_1) \geq \rho_1\bar{v} + ah\rho_1\bar{v}$$

$$\lambda_{\min}(W_2) \geq a\rho_0\bar{v} + a\rho_1\bar{v}$$

NUMERICAL RESULTS

In this numerical example, the calculation of the robustness bounds ρ_0 and ρ_1 for the parameter uncertainty matrices Δ_0 and Δ_1 of the closed-loop system (25) with a PI-Posicast controller is proposed, applying Theorem (Kharitonov, 2012). First, the robustness bounds for the norms of the perturbed matrices $\|\Delta_0\|$ and $\|\Delta_1\|$ are calculated according to Theorem. Then, a robustness condition is determined for disturbances in a parametric space ($\Delta\zeta$, $\Delta\omega_n$). The fixed values for the second-order plant are $\zeta = 0.2$ and $\omega_n = 1$. Given the previously tuned PI-Posicast controller parameters $h = 3.2064$, $k = 0.6550$, $k_p = 0.1810$ and $k_i = 0.2742125$, the norms of the parameter uncertainty matrices (27) and (29) must satisfy: $k_i = 0.2742125$. Then, the norms of the parameter uncertainty matrices (27) and (29) must satisfy:

$$\|\Delta_0\| < \rho_0 \tag{37}$$

And:

$$\|\Delta_1\| < \rho_1 \tag{38}$$

The matrices A_0 and A_1 defined in equations (16) and (17) are:

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.1796 & -1.1185 & -0.4 \end{bmatrix} \tag{39}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -0.0946 & -0.0624 & 0 \end{bmatrix} \tag{40}$$

The matrix $W = W_0 + W_1 + hW_2 > 0$ is chosen as the identity matrix for computational simplicity in the calculation of the Lyapunov matrix, so the matrices $W_0 > 0$, $W_1 > 0$ and $W_2 > 0$ are defined as:

$$W_0 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \tag{41}$$

$$W_0 = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \tag{42}$$

$$W_0 = \begin{bmatrix} 0.0624 & 0 & 0 \\ 0 & 0.0624 & 0 \\ 0 & 0 & 0.0624 \end{bmatrix} \tag{43}$$

For the perturbed system (30) to be stable, conditions 1, 2, and 3 of Theorem (Kharitonov, 2012) must be satisfied. Then, using the semi-analytical method outlined in the definition, the maximum norm of the Lyapunov matrix $U(\theta)$ in the interval $[0, h]$ is obtained:

$$\bar{v} = \max_{\theta \in [0,h]} \|U(\theta)\| = 10.3225 \tag{44}$$

In turn, the norm induced by the Euclidean norm of the matrix is:

$$a = \|A_1\| = 0.1133 \tag{45}$$

The three conditions of Theorem (Kharitonov, 2012) yield three inequalities with two unknowns for the upper bounds of robustness (equations (31) and (32), respectively). To determine the value of these bounds for the unknown matrices (27) and (29) that form the perturbed system (30), we apply the condition (35). This allows us to find the upper bound of the norm $\|\Delta_1\|$ as follows:

$$\rho_1 = 0.003$$

This indicates that the maximum value of the norm of the matrix Δ_1, which contains the perturbed parameters $\Delta\zeta$, $\Delta\omega_n$, and must be $\|\Delta_1\| \leq 0.003$. Now, to calculate the robustness bound for matrix Δ_0, we ensure that conditions (34) and (35) are met simultaneously, then:

$$\rho_0 = 0.001$$

From the above, it can be concluded that the upper bounds of robustness, denoted as ρ_0 and ρ_1, for the norms of the disturbed unknown matrices $\|\Delta_0\|$ and $\|\Delta_1\|$, are given by the calculated values. The parameters h, k_i, k_p, k, ζ and ω_n are fixed, which allows us to obtain the conditions of Theorem (Kharitonov, 2012), and these conditions are satisfied. Therefore, the perturbed system (30) is exponentially stable with the fixed parameters $\|\Delta_0\| \leq 0.001$ and $\|\Delta_1\| \leq 0.003$. Figure 6 shows the convergence of the states $e(t)$, $\dot{e}(t)$ and $\ddot{e}(t)$ to their equilibrium points for the disturbed closed-loop system (30).

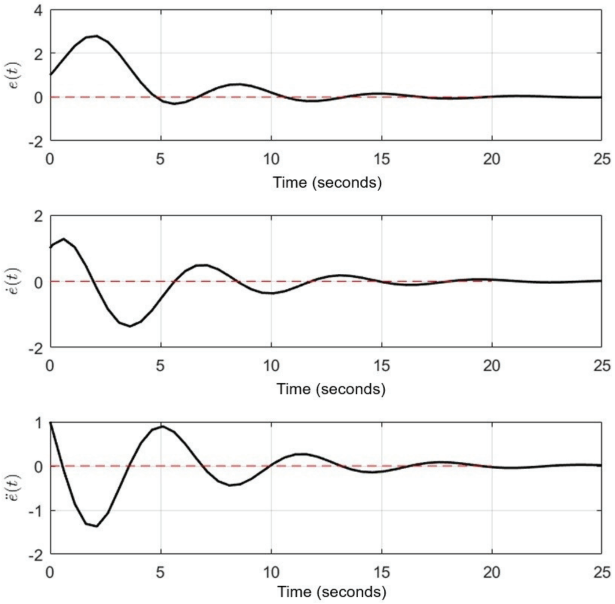

Figure 6. States of the second-order system perturbed with Robust PI-Posicast controller.

If a robustness condition is desired in terms of the parametric disturbances $\Delta\zeta$ and $\Delta\omega_n$, then the expressions of the matrices Δ_0 and Δ_1 must be considered in the robustness conditions (31) and (32). By using the induced norm, we can rewrite conditions (31) and (32) as follows:

$$\left\|\Delta_0\right\| = \lambda_{\max}\left(\Delta_0^T\Delta_0\right) \le \rho_0^2 \tag{46}$$

$$\left\|\Delta_1\right\| = \lambda_{\max}\left(\Delta_1^T\Delta_1\right) \le \rho_1^2 \tag{47}$$

The above implies:

$$4\zeta^2\Delta\omega_n{}^2 + 8\zeta\Delta\zeta\Delta\omega_n\left(\Delta\omega_n+\omega_n\right) + 4\Delta\zeta^2\left(\Delta\omega_n+\omega_n\right)^2$$

$$+\Delta\omega_n{}^2\left(\Delta\omega_n+2\omega_n\right)^2$$

$$+\Delta\omega_n{}^2 k\left(\Delta\omega_n+2\omega_n\right)^2\left(k\left(k_i^2+k_p^2\right)+2k_p\right) \le \rho_0^2 \tag{48}$$

$$\Delta\omega_n{}^2\left(k-1\right)^2\left(\Delta\omega_n+2\omega_n\right)^2\left(k_i^2+k_p^2\right) \le \rho_1^2 \tag{49}$$

Although an explicit condition cannot be derived from equations (48) and (49) that shows the set of values for the disturbances $\Delta\delta$ and $\Delta\nu$ for which conditions (34), (35), and (36) are satisfied, it is possible to numerically determine the region of the parameter space where equations (37) and (38) are satisfied.

$$\left\|\Delta_1\right\| = 4.8201\times10^{-4} \le \rho_1 = 0.003$$

$$\left\|\Delta_0\right\| = 1.8193\times10^{-5} \le \rho_0 = 0.001$$

Figure 7 illustrates the parameter space ($\Delta\zeta$, $\Delta\omega_n$)where the system uncertainty parameter (30) remains stable. The figure highlights a specific ordered pair chosen within the stability region ($\Delta\zeta = 0.0002$, $\Delta\omega_n = 0.0009$). By selecting this point, it is observed that the corresponding values of the matrices Δ_0 and Δ_1 satisfy the robustness conditions (31) and (32).

This indicates that, for the selected pair of disturbances values, the system preserves exponential stability according to the conditions established by Kharitonov's theorem (2012). The corresponding matrices are evaluated for this parameter pair, confirming that the system remains within the stability bounds defined by the robustness criteria. Consequently, the chosen parameter space defines the domain of attraction in which the perturbed system continues to operate stably despite the presence of parameter uncertainty.

In Figures 4 and 5, obtained through the frequency-domain approach, it is observed that the values of the parametric disturbances for which the closed-loop system with the PI-Posicast controller remains stable are $\Delta\zeta \in [0, 0.45]$ and $\Delta\omega_n \in [0, 5]$. In contrast, Figure 7, obtained through time-domain analysis, presents the parameter space ($\Delta\zeta$, $\Delta\omega_n$) where the closed-loop system (3.49) remains stable. The figure demonstrates that the system is stable as long as the values of the parametric perturbations are within the specified bounds for $\Delta\zeta \in [-0.0004, 0.004]$ and $\Delta\omega_n \in [-0.021, 0.021]$.

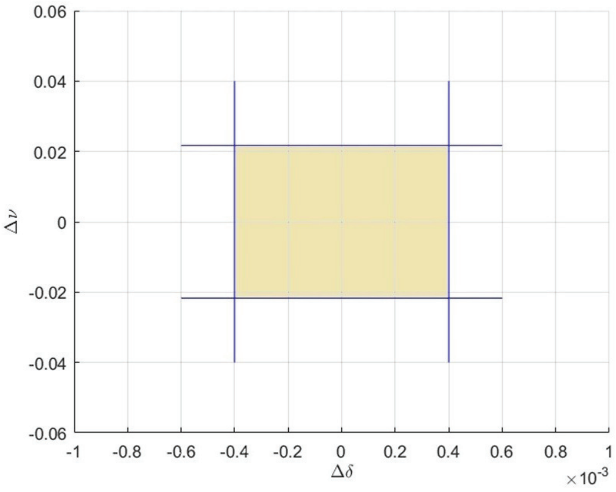

Figure 7. Parameter space ($\Delta\zeta$, $\Delta\omega_n$) that keeps the perturbed system stable.

Comparing the results from both approaches reveals that the robustness bounds obtained from the time-domain analysis, using full-type Lyapunov-Krasovskii functionals, are significantly more conservative than those derived from the frequency-domain analysis. This indicates that, while the frequency-domain method provides more relaxed stability conditions, the time-domain approach imposes stricter criteria, guaranteeing a higher level of stability at the expense of conservativeness.

Conclusions

In this work a Proportional-Integral (PI) control loop was incorporated to enhance the robustness properties of the system. This combined PI-Posicast control strategy aims to improve system performance, particularly in terms of stability when subject to parametric uncertainty.

A comprehensive robustness study was conducted in the time domain using full-type Lyapunov-Krasovskii functionals, focusing on robustness bounds for additive perturbations in the plant parameters. Unlike the frequency-domain approach, which provides only relaxed stability conditions, the time-domain analysis offers a more rigorous and reliable assessment of system robustness under parameter uncertainty. This is because Lyapunov-Krasovskii functionals capture the dynamics of delayed and perturbed systems directly in the time domain, allowing for stability guarantees that are stronger and more meaningful in practice.

Furthermore, the robust stability of the PI-Posicast controller applied in cascade to the underdamped second-order system was analyzed. It was shown that the system remains robust to additive disturbances in the nominal plant parameters, ensuring stability across a range of uncertainty values. The robustness levels were calculated, providing explicit bounds that guarantee closed-loop stability even in the presence of parametric perturbations. This was achieved by applying Lyapunov-Krasovskii-type functionals, which enabled the identification of critical uncertainty limits beyond which the system would lose stability.

Importantly, the analysis indicates that when the plant parameters approach the calculated stability boundaries defined by the uncertainty limits, the system exhibits a reduced margin of stability. In such cases, the system's poles shift closer to the imaginary axis in the complex plane, reflecting a slower decay rate and a lower damping ratio. Conversely, when the parameters remain well within the stability region, the poles are farther from the imaginary axis, resulting in faster convergence and stronger robustness.

In conclusion, the proposed PI-Posicast controller not only improves the stability of the underdamped second-order system but also provides clear robustness guarantees in the presence of parametric uncertainty. The results demonstrate that time-domain analysis using Lyapunov-Krasovskii functionals is superior to frequency-domain methods, as it ensures stricter and more reliable stability margins under uncertainty. This approach highlights the trade-off between conservatism and practical safety, showing that the time-domain framework is more effective for guaranteeing robust stability in systems subject to parameter uncertainty. Future work could explore further refinements in the controller design to achieve an even better balance between performance and conservatism.

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