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Design and evaluation of a free-space optical communications link using pulse width modulation (PWM)

Diseño y evaluación de un enlace óptico por espacio libre basado en modulación de ancho pulsos (PWM)

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Abstract

Free-space optical communications (FSO) use light as information carrier. Light travels through atmosphere or vacuum and reaches a distant receiver. Light is generated by semiconductor lasers (LDs) or light-emitting diodes (LEDs) and can be modulated by varying the injection current on these devices. To carry information, basic modulation techniques are analog as intensity (IM), amplitude (AM), frequency (FM) or phase (PM). However, pulse time modulation (PTM) techniques as pulse-width modulation (PWM) or pulse-position modulation (PPM) are more performant. In PWM, information is imprinted on the duration of a pulse train; PPM codes information on the relative position of narrow pulses of fixed duration. Analog and PTM formats are widely used over optical fiber transmissions. However, works using PWM over FSO communications are not abundant in technical literature. As FSO links represent an alternative for "last-mile" communications (up to 4 km), analog or digital information, such as remote sensing, monitoring and measuring instrumentation signals, environmental monitoring sensing, video and voice, etc., can be efficiently transmitted using lowcomplexity and low-cost hardware. Under this perspective, this paper reports the design, realization and laboratory evaluation of a prototype of short-distance (up to 1 km) FSO link based on optical PWM. The link operates at a wavelength of 1.5 µm and transmits information in a 10 MHz range, as has been tested in short distances in an indoor laboratory environment. The system is based on in-home developed PWM using commercial off the shelf (COTS) optics and electronics. The beamforming optics subsystems use inhome fabricated lenses. The transmitter delivers a 2.5 mW collimated beam. The receiver collects a fraction of the transmitted power and feeds fiber-pigtailed photodetectors. To date, the optical link has been evaluated over 5.5 m. Sensitivities of 2.5 and 0.5 μ W for PIN and APD based photoreceivers are achieved and ensure high quality information recovery.

Keywords: Free-space optical link (FSO), optical modulation PWM modulation, laser diodes, PIN photodetectors, APD photodetectors.

Resumen

Las comunicaciones ópticas por espacio libre (FSO) emplean luz para transportar información. La luz viaja por atmósfera o vacío hasta un receptor distante. La luz se genera mediante diodos laser (DL) o diodos emisores de luz (DEL) y puede modularse variando la corriente de inyección en ellos. Para portar información, las técnicas analógicas básicas son modulación de intensidad (IM), amplitud (AM), frecuencia (FM) y fase (PM). Sin embargo, técnicas de modulación temporal de pulsos (PTM) como modulación de ancho de pulso (PWM) o modulación por posición de pulsos (PPM) son más eficaces. PWM codifica información en la duración de un tren de pulsos; PPM en la posición relativa entre pulsos de duración fija. Las modulaciones analógica y PTM se utilizan ampliamente en transmisiones vía fibra óptica. Sin embargo, trabajos relativos al uso de MTP en FSO son escasos en la literatura especializada. Los enlaces FSO representan una alternativa para comunicaciones de última milla (hasta 4 km) para transmitir información analógica o digital como sensores remotos, señales de instrumentación, monitoreo ambiental, video y voz, etc., mediante equipos de baja complejidad y bajo costo. En esta perspectiva, este trabajo reporta el diseño, realización y pruebas en laboratorio de un prototipo de enlace óptico basado en PWM para distancia hasta 1 km. El enlace opera con longitud de onda de 1.55 μ m y ancho de banda de 10 MHz. El transmisor entrega 2.5 mW de luz colimada. El receptor colecta una fracción de la potencia transmitida y alimenta fotodetectores fibrados. A la fecha, el enlace óptico se ha evaluado en un enlace de 5.5 m. La fotorecepción con fotodiodos PIN y avalancha (APD) presenta sensitividades de 2.5 y 0.5 μ W, respectivamente. La información se recupera con alta calidad.

INTRODUCTION

A free-space optical (FSO) communication system uses light as information carrier. Light travels from a transmitter to a distant receiver through a propagating media such as the terrestrial atmosphere or the vacuum in the open space (Pratt, 1969; Gowar, 1993; Hemmati, 2009; Ghassemlooy *et al.*, 2013). Optical communications systems using light as information carrier are being studied and developed since several decades in countries of advanced technology capabilities.

Nowadays, advanced optical communications systems in the telecommunications industry worldwide use mainly optical fibers as the transmission media. An alternative to optical fiber cables is using free-space transmission through the terrestrial atmosphere or the free space outside of our planet. The interest of developing FSO links for communications started in the 1960-1970 decade and it has been maintained along the last decades for potential applications of space communications and also terrestrial links. FSO is an alternative to wireless radio systems for indoor and outdoor communications between buildings or open spaces (Gowar, 1993; Sander et al., 1986; Khalighi & Uysal, 2014; Sadiku et al., 2016, Raj et al., 2023, Jahid et al., 2022; Majumdar, 2015; Kaushal et al., 2017; Hemmati, 2020; Lambert & Casey, 1995).

In optical communications, either using optical fibers or free-space, the basic modulating techniques modify the intensity (IM), the phase (PM) or the frequency (FM) of the carrier light. In FSO links, the preferred modulating techniques are based on either, digital pulse modulation (on-off-keying, OOK), or pulse-time modulations (PTM) (Jaid *et al.*, 2022; Majumdar, 2015; Kaushal *et al.*, 2017; Hemmati, 2020; Lambert & Casey, 1995). PTM techniques are mainly represented by pulse width modulation (PWM) and pulse position modulation (PPM). PWM and PPM are high performant modulation formats and widely used (Ebrahimi *et al.*, 2018). These techniques are relatively easy to implement as the hardware is low-complexity and low cost when compared, for instance, to OOK digital format.

A brief comparison of some operating basis between OOK digital modulation and analog PWM is given here. Number of authors (Alimi *et al.*, 2024; Kaushal *et al.*, 2017, Ghassemlooy *et al.*, 2013) state that OOK is the most basic direct optical intensity modulation. In this case, digital data consisting of 0s and 1s modulate the optical emission between two optical power levels. A low level corresponds to 0s and a high level represents 1s of binary data. OOK, however, requires a complex process of analog to digital and digital to analog conversions for transmitting and recovering analog information. OOK transmission requires a bandwidth in proportion to its bit rate (sampling rate times coding bits number). After transmission, OOK pulses arrive attenuated to the receiver and should be regenerated. OOK requires of a clock recovery step by processing the received data. Synchronization between received data and recovered clock is required for ensuring an efficient decision threshold, which determines the bit error rate (BER). BER, which depends on the ratio of bit energy to noise power, is the quality measure of OOK transmissions.

PWW is a hybrid modulation between purely analogue and strictly digital techniques. The analog information is imprinted not on the amplitude of a carrier (as in pulse amplitude modulation, PAM) but on the duration of the high state of a pulse train. The transmission efficiency depends on the optimum sampling of the analog signal, i. e., a sampling rate of at least twice the maximum frequency of the analog signal. PWM waveform is realized by comparing a sampling saw-tooth waveform and the analog signal by using an electronic comparator. Comparator generates a PWM waveform. The spectrum of the PWM waveform contains the baseband modulating spectrum and also harmonics of the baseband around harmonics of the sampling saw-tooth carrier. The bandwidth requirement of PWM is 2/3 related to NRZ-OOK bandwidth (Ebrahimi et al., 2018). For optical transmission, the PWM is converted to optical pulses. At the receiver frontend, optical pulses arrive normally strongly attenuated. While the receiver is able to regenerate the received pulses, information can be efficiently recovered as it is contained on the duration of the received pulses. PWM transmissions are evaluated by the signal to noise ratio (SNR) at the output of the receiver.

A comparative summary of OOK and PWM operating principles are listed in Table 1.

As it is well known, PWM codes and transmits either analog or digital signals, such as video, voice and data, not on the amplitude but rather on the duration of an electrical pulse (Fan & Green, 2007; Gutierrez, 1990; Ghassemlooy & Wilson, 1994; Gutierrez & Torres, 1996; Santiago et al., 2012). For optical transmission, the time modulated electrical pulses are converted to equivalent optical pulses, which are transmitted to the distant optical receiver. As the optical pulses travel through the free space, they become attenuated on depending on the transmission distance. However, at the receiver, the optical pulses, converted to electrical, can be regenerated and the information signal is recovered by simply low-pass filtering the modulated pulses. PWM shows an improved performance when compared to simpler intensity or amplitude modulation. Improved trans-

OOK	PWM
Complex circuitry for sampling analog information, quantization and digital coding (A/D conversion) at the transmitter	Low complexity circuitry for generating a saw-tooth carrier. PWM waveform is generated by comparing the carrier and the analog signal
Wide-bandwidth requirements in proportion to data bit rate. Bit rate = (sampling frequency)(number bits)	Bandwidth includes baseband spectrum and harmonics of the sampling frequency. Practical bandwidth is limited up to third or fourth harmonic s of the sampling frequency
Binary data directly modulates the optical power to provide an optical data stream	PWM waveform modulates the optical power. PWM optical pulses are transmitted
At the receiver optical pulses arrive attenuated and data regeneration is required	At the receiver PWM optical pulses arrive attenuated and regeneration is required
Clock recovery from digital data is necessary for data synchronization and efficient information recuperation after a digital to analog conversion step	Transmitted information is recuperated from regenerated PWM by simple low-pass filtering step
OOK performance is evaluated by the bit error probability BER	PWM performance is evaluated by the signal to noise ratio $(S / N =)$ at the output of the receiver.

Table 1. A comparison between OOK and PWM operatin principles

mission quality comes from the fact that information is imprinted on the duration of the pulses instead of their amplitudes.

By reviewing the technical literature, it has been found that PWM-based implementations of FSO communications are not abundant in the technical literature. Most reported uses of PWM are related to transmissions of video over optical fiber channels (Fan & Green, 2007; Gutierrez, 1990; Ghassemlooy & Wilson, 1994; Gutierrez *et al.*, 1996; Santiago *et al.*, 2012). By prospecting the use of PWM for free space optical communications, very recent work (Ding *et al.*, 2023) describe an optical communication relay system to realize communication networking where the binary data are converted into PWM signals. Other relative recent works, involving PWM, report the conversion of OFDM to PWM modulations in applications of visible light communications (Zhang *et al.*, 2017; Ebrahimi *et al.*, 2018).

In Mexico, the optical communications field is scarcely studied and developed. Some efforts on research and development are mainly conducted in academic institutions. Optical systems technologies can be developed aiming to increase our scientific and technical capabilities and associated knowledge. At INAOE, developing free space optical links is one of the research activities and acquiring the "know how" is a fundamental issue. Distances around 1-2 kms can be refered as the equivalent of the "last mile" reference for short range optical links. Given our interest of acquiring the "know how" for developing free space optical links and also demonstrating the factibility of low cost implementations using commercial of the shelf (COTS) components, the distance of 1 km has been considered as a starting distance. Our developing efforts, which are based on a low cost semiconductor lasers emitting optical powers of some milliwatts (1-3 mW) and also on developing our beamforming optics, are able for testing our first experimental link over 1 km.

In this paper, the set-up of a prototype of FSO link for transmitting information through the atmosphere is described.

Potential applications aim communicating interbuilding or near-distance facilities such as educational campuses, enterprises or residential areas over transmission distances up to 1 km. The proposed free space optical link has been designed and implemented as a low cost prototype using COTS components. In this paper, our main goal is demonstrating the feasibility of acquiring and developing these technologies for practical applications of FSO links. At this step, even if the transmission of analog information by either, radio or optical media seems "outdated", the demonstrative application of our scheme is the transmission of analog signals and NTSC video for validating the developed capabilities. However, our scheme is able for transmitting both analog and digital data in a band up to 10 MHz. It is well known that efficient sampling of an analog signal requires of reading it at a rate of at least, or greater than twice its maximum frequency. This means that an analog video signal, whose maximum frequency is 6 MHz, can be efficiently sampled at a rate of 12 MHz. However, when transmitting video, it is recommended a sampling rate of three times the color carrier frequency, which avoids undesired visual beats on the reconstructed video signal (Ghassemlooy & Wilson, 1994). That is the reason why on the proposed free space optical link, the sampling rate is 25 MHz, which is enough for sampling analog signals in a bandwidth of 10 MHz and also ensuring optimum over-sampling of the "outdated" analog NTSC video signal.

As an analog transmitting scheme, our optical link can be used the transmission of video and images for surveillance or visual monitoring of natural phenomena or man-made process of varied nature. It can also be used for transmitting analog instrumentation signals coming from sensors and actuators of physical variables such as temperature, pressure, liquid levels, etc., which are generated as analog voltages or currents. Even if our system only shows analog transmission, it is also able for transmitting digital signal such as on-off keying (OOK) of base band digital formats. Later, the research will enable further studies and development of more complex modulation and transmission schemes for free space optical links.

The proposed optical link is configured by a transmitter side, which includes an electronics subsystem and transmitting beam forming optics using lenses for concentrating the light that is sent to a distant receiver. The proposed FSO link uses a low cost semiconductor laser and light is delivered through a standard single-mode optical fiber (SSMOF). The emitted optical power is 2.5 mW (+4 dBm) at a wavelength of 1.55 µm. At the transmitter side, before the beamforming optics, an optoelectronic section generates an electrical subcarrier which is modulated by the information signals. At the far side, a receiving optical subsystem collects a fraction of the transmitted light and concentrates it into a SSMOF pigtailed photodetector. Once the received light is photodetected, amplified and processed, the carrier is demodulated for recuperating the transmitted information.

In this paper, the proposed FSO link is realized by revisiting and using a PWM optical carrier. The developed scheme has been successfully tested indoors our laboratory.

In the next sections of this paper, the design, realization and laboratory validation of the FSO prototype are described. The configuration of the transmitting and receiving beam conforming optics, the FSO power budget, the PWM generation and demodulation, the experimental set-up and characterization of the free space optical link are described successively. The scheme has been tested in laboratory for evaluating its performance when using different photoreceiver configurations. Optical transmission losses are emulated by attenuating the transmitted light. An optical power range between 50 to $0.1 \,\mu$ W is received and demodulated for determining the performances and sensitivity of the optical link.

PROPOSED FREE-SPACE OPTICAL TRANSMISSION LINK

The proposed FSO link is shown in Figure 1. It is configured by two complementary subsystems. The first one is the transmitter, which is integrated by an optical source and the modulating electronics (a PWM generator and the laser driving electronics). The PWM optical beam is then passed through the transmitting beam conforming optics (a set of lenses), which collimate the laser beam in the direction of a distant optical receiver. The collimated optical beam travels through the atmosphere. After a given distance, light is collected by the receiving conforming optics, which concentrates the light into the core of a single mode fiber pigtailed phototedector. The generated photocurrent is then converted to an equivalent voltage. The voltage is amplified and the pulses are regenerated. The reconstructed PWM signal is demodulated by a low-pass filter and the information signal is recovered.

The strength of pulse time modulations such as PWM or PPM is represented by the capability of regenerating the received pulses before recovering the modulating information. In this work, it is demonstrated that the information is successfully recovered when the power of the received light ranges between 50 and 0.1 μ W. This range is determined, in the upper limit (50 μ W) by the saturation of the optical receiver; the lower limit depends on the sensitivity of the photoreceiver. The photoreceiver has been implemented by using two types of photodetectors: positive-intrinsec-negative (PIN) photodetector and avalanche photodetector (APD) (Gowar, 1993). The performances of the optical link using the two types of photodetection are compared and evaluated. It has been found that the PIN-based photoreceiver is the least performant; the best performance is obtained by the APD-based photoreceiver.



Figure 1. The proposed free-space optical link based on PWM

BEAMFORMING OPTICS FOR FREE-SPACE TRANSMISSION

A key aspect for configuring a FSO link is the design and implementation of transmitter and receiver optics. At the transmitter side, the optical beam comes from the output face of a SSMOF. The emitted beam normally starts to diverge after going out from the optical fiber and requires being collimated before FSO transmission. For free-space practical transmission, beamforming optics using lenses at the transmitter and receiver sides are implemented as shown by Figure 2 (Hudson, 1969). The transmitting beamforming optics consists of a set of concave-convex and biconvex lenses which focusses the diverging laser emission onto a highly collimated beam. The optical beam travels through the free-space and normally will diverge, covering an increasing area, on depending on the propagating distance. At the distant side, the receiving beamforming optics consists of a set of three lenses (biconvex-biconcave-spherical), which collect a fraction of the radiated optical power and focus it to enter in a fiber-pigtailed photodetector.

The beamforming optics were implemented and characterized in a laboratory testbed over a transmission distance of 5.5 m. The optical characteristics are listed in Table 2. At the transmitter side, the optics collimates the 2.5 mW laser power onto a light beam of 8 mm-diameter. On the testbed, the transmitted optical signal was attenuated for receiving and detecting the minimum power levels that determine the photoreceiver sensitivity. The sensitivity is the minimum power that can demodulated for recovering the transmitted information. In this way, the transmitted power was progressively attenuated and it was observed that optical powers higher than 50 μ W overload the photoreceiver. As already mentioned, the transmitted signal was attenuated to receive a 50 to 0.1 μ W power range.



Figure 2. The free-space optics transmitter and receiver subsystems

Technical literature (Kaymak *et al.*, 2018; Alimi & Monteiro, 2024), reports that FSO communications require of maintaining accurate alignment and pointing between transmitter and receiver. Pointing and alignment errors are determined by diverse facts. One of such

Table 2.	Beamformir	g optics	characteristics
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facts is related to the size of the beam footprint at the receiver of the free space optical link. The footprint size depends on the geometrical losses that the beam suffers through the propagation distance. These losses are determined by the divergence angle of the optical beam, the distance between transmitter the receiving area and losses by atmospheric variations (fog, rain, scintillation, density, etc.) Using a narrow beam limits the beam divergence thus increasing the received power density. FSO links often use beams showing divergence angles in the range of some milliradians. This condition often requires of a precise alignment, which is improved by using a pointing and tracking system.

According to the transmitting optics parameters listed in Table 2, when considering the propagation of a Gaussian wavefront (Gowar, 1993), at the transmitter output, the divergence angle is around 14 millirad. This angle determines a footprint size at the receiving plane of about 0.05 m². The footprint is much larger than the optics receiving aperture (0.001 m²). In this way, it can be stated that the proposed FSO link does not require strictly of a pointing and tracking system for transmission over a distance of 1 km.

FREE-SPACE OPTICAL LINK POWER BUDGET

The proposed optical link uses an optical source emitting light at 1.55 um and average optical power of 2.5 mW. The optical power delivered at the output of the fiber pigtailed laser is collimated by the beamforming optics, which performs as an optical antenna. Such an antenna exhibits a gain, which depends on the area of the transmitting lens and on the optical wavelength. The transmitted light beam diverges while propagating through the free-space and the power density decreases on depending on the traveled distance. The decrease of power density represents free-space losses. At the distant side, the receiving optics, acting also as optical an-

Transmitting optics	concave-convex lens	biconvex lens	Lenses array
Diameter (Φ)	35 mm	45 mm	
Thickness (d)	9.41 mm	10.7 mm	Distance between lenses = 4 mm
Refractive index (n_{lens})	1.500	1.500	$N_{air} = 1.00021$
Focal distance (f)	52.9 mm	53.4 mm	
Receiving Optics	biconvex lens	biconcave lens	
Diameter (Φ)	37 mm	5.4 mm	
Thickness (d)	14.5 mm	1.5 mm	Distance between lenses = 42.5 mm
Refractive index (n_{lens})	1.500		$N_{air} = 1.0$
Focal distance (f)	54.3 mm	- 6.3 mm	

tenna, collects a fraction of the transmitted power density and delivers it as an optical signal to the photoreceiver. The received optical power is determined by the power budget of the optical link and depends on the optical power from the laser source, the transmitter and receiver optical gains and the overall propagating optical losses (Gowar, 1993).

For transmission purposes, the optical signal can be modelled as a Gaussian optical beam coming out from the core of a single mode optical fiber of circular section and 10 µm diameter. After leaving the optical fiber, the optical beam immediately starts diverging. The transmitter optics collects the diverging light and concentrates it as a collimated optical beam of 8 mm diameter. The collimated light is launched to propagate through the free-space. As the optical beam propagates, it will continue to diverge and the optical power will cover a wider area on depending on the transmission distance. At the receiver, the area of the collecting optics will concentrate a fraction of the transmitted optical power into a photoreceiver. The received optical power must be higher than the sensitivity of the photoreceiver for a successful free-space optical link.

The transmission power budget can be calculated in order to estimate the maximum transmission distance as the scheme is limited by the available optical power. From the wave propagation theory through the free-space (Sander *et al.*, 1986; Gowar, 1993), the received optical power (Pr) is given as:

$$\boldsymbol{P}_{r} = \boldsymbol{P}_{t} \ast \boldsymbol{G}_{t} \ast \boldsymbol{G}_{r} \ast \boldsymbol{L}$$
(1)

Where:

 P_t = optical power from the laser source

*G*_t and *G*_r = the gains of the transmitting and receiving optics, respectively

L = the propagation losses

The transmitting and receiving optical gains are given as:

$$\boldsymbol{G}_{t,r} = \frac{4\pi \boldsymbol{A}_{t,r}}{\lambda^2} \tag{2}$$

Where A_t and A_r are the areas of the transmitting and receiving optics, respectively and λ is the optical wavelength.

The propagation losses are given as:

$$\boldsymbol{L} = \left(\frac{\lambda}{4\pi \boldsymbol{I}}\right)^2 \tag{3}$$

Where *l* is the link distance.

The transmission-reception parameters which determine the power budget are listed in Table 3.

After (1) trough (3), in a first order calculation without considering atmospheric effects such as absorption, dispersion, rain, fog, etc., the optical power budget for a free-space link covering a distance of 1 km indicates that the received optical power is about 100 μ W.

In the power budget of the proposed FSO link, optical coupling losses at the transmitting and receiving optics are not considered. Actually, at the transmitter, coupling losses are very low as all the laser power at the output of the emitting optical fiber is pickep-up by the transmitting optics, which ensures a collimated laser beam. The optical power has been measured at the output of the optical fiber and at the output of the collimating optics. The measured optical power shows no variation.

At the receiving optics high coupling losses are introduced by the optical adapter to fiber optics. Coupling losses mainly depend on the misalignment between the biconcave lens and the spherical interface of the fiber optics adapter. The coupling losses are equivalent to one-third of the power delivered by the biconcave lens. This means that in the short distance characterization of the proposed FSO link, the phototetector receives only one-third of the optical power as collected by the front surface of the receiving optics.

Related to the laser beam propagation through the atmosphere, besides geometrical losses, light will experiment attenuation, blockage, absorption, scattering, dispersion, scintillation, turbulence, etc. (Jaid *et al.*, 2022; Ahmed *et al.*, 2020; Ghassemlooy *et al.*, 2013) These loss factors, caused by the atmospheric components

Table 3. Optical parameters for free-space optical transmission

Laser optical power (mW)	Optical wavelength (µm)	Transmitting optics diameter (mm)	Receiving optics diameter (mm)	Transmission distance (m)	Propagation losses	Received optical power (µW)
2.5 (+ 4 dBm)	1.55	10 (G = 86 dB)	40 (G = 98 dB)	1000	198 dB	100 (- 10 dBm)

and atmosphere dynamics are unavoidable in real operation of FSO links.

Studying and analyzing atmospheric effects on the proposed FSO link is out of scope at this stage of development. It is worth noting that studying the atmospheric performances is mainly based on statistical models. Absorption is for instance, studied by Rayleigh or Mie models that provide estimations of the effects of atmospheric molecules and particles on the light propagation.

Turbulence is a complex factor in the atmosphere. It is mainly related to atmospheric refractive index variations, as caused by temperature, wind, whether, etc. Turbulence mainly introduces power fluctuations or beam deviations causing distortion or loss of link availability. Turbulence is analyzed by statistical models, such as Log-Normal, Gamma-Gamma or K distribution. Turbulence can be mitigated by elaborated receivers using, for instance, adaptive optics.

The impact of turbulence on the proposed FSO link seems no critical at short distances as proposed until now. However, the effects on long distance transmissions require of studying and modeling turbulence in order determine feasible solutions.

Before real field testing, the proposed free-space optical link has been characterized in laboratory environment. The testing setup consists of a 5.5 m optical link and the transmission-reception of analog information signals has been evaluated. As already described in the introduction of this paper, the electronics subsystem is a low-cost proprietary design of a PWM transmitter and receiver circuits. The test information signals are analog signals and video, in a bandwidth of 10 MHz. The main purpose of the laboratory evaluation is to determine the sensitivity limits of the photoreceiver in order to ensure an optimum free-space transmission-reception process for link distances up to 1 km.

In the next section of this paper, the implementation of the PWM optoelectronic scheme for a free-space transmission is described.

Free-space optical transmission using a pulse width modulated (PWM) electrical subcarrier

Optical communications links use laser or led light at optical wavelengths that can be transmitted through electromagnetic transparent physical media. In the case of free-space optical links, terrestrial atmosphere and vacuum are highly transparent to optical wavelengths longer than 1 μ m. In the frame of the work reported here, our optical link operates at a wavelength of 1.55 μ m. As light performs as information carrier, it should be modulated for transporting information signals. As already stated in the introduction of this paper, laser

light can be modulated by varying the injection current and in this way, information signals are imprinted on the light carrier. Pulse-time modulations, such as PWM and PPM, are performant techniques and low-complexity for FSO transmissions as compared to digital on-off keying (OOK) modulations (Khalighi & Uysal, 2014). Our experimental free-space optical link uses own-designed PWM electronics. A PWM signal is electrically generated by the block diagram depicted in Figure 3. The transmitting section of a PWM is based on the generation of a square-wave carrier. The square wave is then integrated for generating a saw-tooth waveform. This last waveform is compared to the modulating signal. The output of the comparator is a square waveform where the modulation is carried on the duration of the successive pulses. Figure 4 illustrates the signals inherent to the generation-modulation process. The modulated square-wave subcarrier is generated by comparing the amplitudes of the saw-tooth waveform r(t) and the modulating signal m(t), in a process called natural sampling (Black, 1953; Gutierrez, 1990). The comparison is based on two conditions:

- a) While the amplitude of the saw-tooth waveform is higher than the modulating amplitude, a pulse is generated and its duration is determined by the time $r(t) \ge m(t)$.
- b) While the amplitude of the saw-tooth waveform is lower than the modulating signal, no pulse is generated.

To achieve an efficient modulation, the frequency of the pulses train should be at least three times higher than the maximum frequency of the information. In our case, the square wave carrier frequency is 25 MHz and the modulating bandwidth covers 0-10 MHz. The PWM signal is given as:

$$PWM(t) = \begin{cases} 1 & as \quad m(t) \le r(t) \\ 0 & as \quad m(t) > r(t) \end{cases}$$

$$\tag{4}$$

According to Figures 3 and 4, once the PWM signal electrically generated, it is converted to optical pulses by varying the injection current of a laser diode. The transmitting beamforming optics then collimates the laser beam and sends it through free-space. At the receiving side, an optics subsystem collects a fraction of the transmitted power and focuses it to a fiber-pigtailed photodetector. The photodetector is associated to an optical photoreceiver which processes and demodulates the transmitted information. The photoreceiver is based on high-impedance amplifiers. After amplification of the photodetected optical pulses, another electrical comparator regenerates the PWM waveform. The information is recuperated by low-pass filtering the regenerated PWM waveform and the information signal is delivered at the output of the photoreceiver.



Figure 3. Free-space optical link base on PWM subcarrier



Figure 4. PWM signal generation for free-space optical transmission

FREE-SPACE OPTICAL LINK TESING IN LABORATORY

As already mentioned in the introduction section, the implemented FSO link has been tested and characterized aiming to determine the potential coverage for relative short distances, i. e., up to 1 km. As indicated earlier in this paper, the optical power is generated by a fiber pigtailed semiconductor laser emitting an average optical power of 2.5 mW at an optical wavelength of 1.55 µm. The optical transmitter is based on a pulsewidth modulator (PWM), implemented as described in a preceding section. The modulating signal can be either analog or digital, in a 10 MHz bandwidth. At this stage, the optical transmission is evaluated by transmitting analog signals such as sine, tringle and square and analog video waveforms. The amplitude of the test signals is 200 mVpp and frequency of 10 KHz. At the transmitter an optics set-up of lenses collimates the modulated light beam and launches it to free-space for reaching a distant receiver. At the receiver site, light is collected by another receiving beamforming optics, which focus the optical power towards the input of a fiber-pigtailed photodetector. Two different photodetectors are used. A first one is a PIN photodiode; the

second is an APD. Both devices detect the optical power and deliver a photocurrent to a high-impedance photoreceiver.

The FSO link has been tested and characterized indoors, in our laboratory, over a distance of 5.5 m. The link evaluation aims to determine the transmission performance and the receiver sensitivity on depending on the type of the photodetectors. The laboratory tests allowed to determine that the minimum received power for recovering a good quality information signal is about 2.5 μ W when using a PIN photodetector and 0.5 μ W when using the APD. These figures become the reference for testing the proposed optical link.

As the power budget, previously calculated in a preceding section, indicates that after a propagation distance of 1 km, the received level is about 100 μ W, such a power will saturate the photoreceiver. In the laboratory tests, the transmitted optical power is attenuated for an optical reception in a 50 and 0.1 µW range. The upper limit of 50 μ W is determined by the saturation of the photoreceiver. The sensitivity and the quality of the received information signal are evaluated as the merit figures of the proposed optical link. The sensitivity is a parameter determined by the minimum received power that allows a signal to noise ratio for an efficient demodulation of the information signal. The recovered modulating signal presents a signal to noise ratio around 40 dB, when using an APD photodetector, which determines a good transmission-reception quality.

In the following subsections, the performances of the optical link, for each photoreceiver configuration, are described and analyzed.

Optical link evaluation using a **PIN-**high impedance photoreceiver

In a first step, the free-space optical link has been characterized using a photoreceiver based on a PIN and high-impedance amplifier. The PIN photodetector's responsivity is 0.95 at 1.55 μ m and operates up to 2 GHz under a reverse voltage of 20Vdc.

The laboratory experimental testing is realized by receiving the laser beam which travels through the freespace over a distance of 5.5 m. The optical beam is modulated by the PWM subcarrier. The modulating signal is a 10 KHz – 200mVpp saw-tooth waveform. The receiver is adjusted to provide a 1Vpp demodulated signal. The optical link is characterized by measuring the power of the received optical pulses. To simulate freespace losses, at the transmitter side, an optical attenuator regulates the emitted power and the link has been adjusted for receiving a power in a range between 50 and 0.1 μ W. The link performance, when using a PIN photodetector is shown in Figure 5. The black-circles trace represents the amplitude of the received pulses; the red-crosses trace illustrates the amplitude of the recovered modulating waveform. From the graphs, it can be noticed that even if the amplitude of the received pulses decreases, the demodulated signal is recovered with constant amplitude in the analyzed optical power range $(0.1 - 50 \mu W)$. The upper limit is determined by the saturation of the photoreceiver as the optical power becomes higher than 50 μ W. The qualitative evaluation of the optical transmission is illustrated in Figure 6, where the received waveforms are displayed (left-side, optical pulses; right-side, information signals) for received optical powers of 10, 5, 2.5 and 1 µW. After propagation in free-space, the received pulses are not ideally shaped and are determined by the frequency response of high-impedance amplifier (90 MHz). However, the received pulses show the PWM profile, as a wider trace on the trailing edge, which corresponds to the imprinted information signal. To ensure a linear PWM, normally low-modulation index are used ($\Delta t / T_{o}$ < 0.3), $\Delta t = f(V)$ is the pulse width variation, on depending on the modulating voltage V; T_0 is the unmodulated pulse width). From Figure 6, it can be observed that even if the amplitude of the optical pulses decrease, the amplitude of the demodulated signal remains constant, while the received pulses can be regenerated. In this case, when using a PIN photodetector, the sensitivity is about 2.5 µW, and the information is clearly recognizable. For received signal of 1 μ W, the information is still identifiable, but notably noisy.

From the experimental characterization of the proposed optical link, the detection limit of our PIN-based photoreceiver is determined by its noise floor, around 2.5 μ W. Lower power levels cannot be efficiently regenerated and consequently, the information is severely degraded or even lost.

From the reported characterization of our FSO link using PWM, it becomes evident that the modulating information can be efficiently recovered, regardless the attenuation of the optical pulses after the free-space propagation. In the proposed FSO link, the transmitted power is around 2.5 mW (+4 dBm) in a collimated laser beam. As shown in this section, an optical power of the order of 5 µW (- 23 dBm) allows recovering the transmitted information. The difference between the transmitted and the minimum received powers represents an attenuation range of 34 dB when using the PWM technique and PIN Photodetector. This capability represents a unique feature of time modulated pulses such as PWM. The PWM technique compares advantageously to the simpler amplitude modulation (AM) and ensures higher communications quality.



Figure 5. Photodetection based on PIN photodector, black-dots trace corresponds to received optical pulses, red-cross trace shows the demodulated signal amplitude

Optical link evaluation using an avalanche photodetector APD-high impedance photoreceiver

A second testing condition of the proposed free-space optical link is its evaluation and characterization using a photoreceiver implemented by an APD photodetector and a high impedance amplifier. The APD is a photodiode responding to light between 0.9 and 1.6 μ m and responsivity of 0.9 A/W at 1.55 μ m. Its frequency response goes up to 2 GHz under a reverse polarization of 40V. The testing conditions are similar to those already described in the preceding section. The optical transmitter sends the PWM optical carrier. The received optical power is adjusted in a range between 50 and 0.1 μ W. The photoreceiver detects such a range and after demodulating the PWM carrier, it delivers a 1 Vpp demodulated signal.

The free-space transmission is characterized by measuring the received optical pulses and the demodulated information signal. Figure 7 shows the relationship between the optical power of the received pulses and the demodulated signal level when using an APD. In this Figure, the black-circles show that the photoreceiver becomes saturated at optical powers higher than 25 μ W. The response is linear for received powers between 25 and 0.1 μ W. Concerning the demodulated signal level remains constant regardless the decreasing power of the optical pulses.

As an APD photodetector presents optical gain, the receiver sensitivity increases and from this fact, the minimum detected power level decreases as compared to the sensitivity of a PIN photodetector. The sensitivity of the APD-based photoreceiver is $0.1 \ \mu W (-40 \ dBm)$. Similarly to the characterization steps already descri-

Similarly to the characterization steps already described in the preceding section, the received pulses and

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Figure 6. Photodetected signal waveforms corresponding to received optical power based on PIN photodetector: a) 10 μ W, b) 5 μ W, c) 2.5 μ W, d) 1 μ W



Figure 7. Photodetection based on APD photodetector, blackdots trace corresponds to received optical pulses, red-crosses trace shows the demodulated signal amplitude

the demodulated information are displayed in Figure 8. In this case, received signals of 2.5, 1, 0.5, 0.25 and 0.1 μ W, are detected and demodulated. It can be observed that the received pulses are better defined when using the APD-based photoreceiver. The sensitivity level decreases and the pulses and information signals can be demodulated for a received power as low as 0.1 μ W. Even at a power of 0.1 μ W, representing extremely weak optical pulses, the modulation is recuperated although already disturbed by noise.

The detailed characterization of the PWM-based FSO link using either PIN or APD photodetectors, was described in the preceding sections. The performances of the FSO link are summarized in Table 4. It was found that the best sensitivity is provided by using APD. APD improves sensitivity by 10 dB in comparison to PIN-based photodetection.

The proposed FSO link at this stage has been tested by transmitting analog signals. It is well known that analog transmissions are evaluated by the quality of the received signals. Such a characteristic is mainly determined by the signal-to-noise ratio (SNR) at output of the receiver. This is the used criteria for evaluating the performance of our FSO link. To detect and regenerate the PWM-modulated optical pulses, the receiving front-end was configured by a high-impedance photoreceiver based on two different photodetectors; in a first case a PIN photodiode was used. A second configuration, uses an APD. After characterizing the photoreception, optical pulses coming from the free space distance were amplified and regenerated. After regeneration, the PWM modulation is recuperated by lowpass filtering and amplification. It is well known that the noise on a PWM signal depends strongly on the "time jitter", as associated to the rising and falling times of the leading and trailing edges of the PWM waveform (Black, 1953; Di Biasi et al., 1987; Suh, 1987). Low noise is achieved by the fastest rising and falling times. Thus,

high SNR are ensured by the response times of the pulse regeneration at the receiver in order to ensure fast leading and trailing edges of the regenerated PWM. As it has been shown, the performance of the proposed FSO link, when based on a PIN photodetection ensures a SNR of about 30 dB at the output of the receiver. The APD-based photoreceiver provides a SNR is about 40 dB, which represents a higher quality transmission.

The quality of the transmission-reception process is given by the signal to noise ratio (SNR) (Di Biasi *et al.*, 1987; Suh, 1987). SNR is the quantitave figure of the quality of the received signal. SNR is the ratio of the amplitude of the demodulated information to the amplitude of the noise floor at the output of the receiver and is given as:

$$SNR = 20 \log \left[\frac{V_{pp} \ señal}{V_{pp} \ ruido} \right]$$
(5)

The analysis of the SNR of the proposed FSO link is out of the scope of this paper. By measuring the amplitudes of the demodulated signal (1Vpp) and the noise (10mVpp), the SNR at the output of the APD-based photoreceiver is 40 dB, which represents a good quality transmission.

Regarding linearity on analog signal transmissions, classic criteria is inherently associated to the response optical power vs voltage or current on the optical source. Light emitting diodes (LEDs) and laser diodes (LDs), show good linearity at low emitted power but at higher levels, such devices become nonlinear. In this way, linear response between input and output signals depends on the LED or laser power regions. Nonlinear response provokes distorted received signals and undesired harmonics contents in the transmission.

In PWM transmissions, LED or laser nonlinearities are not a critical issue as the optical signal changes between

 Table 4. FSO transmission link performances under short distance laboratory test

Optical power emitted at the output of SSM optical fiber		2.5 mW
Optical power at the o	utput of the optical transmitting lenses subsystem	2.5 mW
Optical power at the ir	nput of the optical receiving lenses subsystem	2.5 mW
Maximum optical power at the input of the photodetector		500 µW
Photoreceiver saturation	≥ 50 µW	
Minimun optical power for a signal to noise ratio of 40 dB of the demodulated signal		5 μW
Optical power thresho	5 μW	
Optical power threshold- APD-based photoreceiver		0.25 μW
SNR	APD-based photoreceiver	40 dB

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Design and evaluation of a free-space optical communications link using pulse width modulation (PWM)



Figure 8. Photodetected signal waveforms corresponding to received optical power based on APD photodetector: a) 2.5 μ W, b) 1 μ W, c) 0.5 μ W, d) 0.25 μ W, e) 0.1 μ W

only two power levels. PWM is similar to digital modulation. The information is not imprinted on the amplitude of the optical pulses but in the time transitions between two power levels. The linear response does not depend on the pulses amplitudes. In this case it is determined by the linear comparison in the PWM modulating process. In PWM, a low modulation index ensures a high linear response (Black, 1953; Suh, 1987). In the proposed FSO link, the duty cycle of the unmodulated carrier is 50 %. A modulation index of 10 % is used, which ensures a high linear response as it is shown in Figures 8 and 9 where input and output triangle waveforms show a linear relationship. Linearity on the proposed FSO link is 98 %.



Figure 9. Photoreceived analog video, on depending on the received optical power 2.5W: a) PIN photodetection of 2.5 μ W, b) APD photodetection of 0.25 μ W

The experimental results show that PWM can be used as a performant technique for transmitting information over short distance optical links. A PWM-based FSO link is a low-complexity and low-cost technique for practical applications. It can be used advantageously for transmitting analog or digital signals for particular purposes such as instrumentation signals, sensing processes, remote monitoring, video and voice and digital data.

To show the capacity for transmitting complex signals, analog video was successfully transmitted. Figure 9a, compares the demodulated video (NTSC color bars pattern) when 2.5 μ W optical power is received by the PIN photoreceiver. Figure 9b shows the same signal when APD detects 10 times less power (0.25 μ W). The higher sensitivity of the APD-based photoreceiver ensures a higher quality transmission, in the order of 10 dB, as compared to the PIN-based reception. Figure 10 shows the live testing of video transmission of the proposed PWM-based FSO link.



tical communicat

Free-space optical communications through the terrestrial atmosphere or vacuum are based on the use of light for transmitting information between point-topoint or point-to-multi-point distant sites. In this paper, the realization and characterization of an FSO link, based on the transmission using a PWM-subcarrier has been reported. The proposed optical link has been designed to transmit information over short distances, in the order of 1 km, for transferring information between buildings, near-remote monitoring facilities, remote surveillance stations and central offices, mobile stations and main facilities, etc. The developed scheme has capabilities for transmitting analog and digital data in a frequency band up to 10 MHz, which enables the transmission of digital or analog information such as voice and video, sensors data, instrumentation data.

CONCLUSIONS

The developed scheme has been designed and configured by low-complexity and low-cost electronic subsystems for generating and demodulating the PWM subcarrier as well as the optical subsystems for transmitting and receiving a free-space propagating optical beam. The scheme has been characterized by using PIN and APD photoreceivers. The best performances have been achieved when using an APD-based photoreceiver. After the power budget calculations, the proposed link will be able to cover a 1 km distance as the received optical power is on the order of 100 μ W. The demodulated signals are recovered with a signal to noise ratio of about 40 dB, which represents good quality transmission for the proposed applications.

The overall optical link has been realized and tested in laboratory as an operating prototype for short distance transmissions. Work for testing the developed system in outdoor environment on distances up to 1 km is in progress. Testing the developed prototype over short distances does not strictly requires of pointing and tracking systems. However, the developed prototype can be easily scaled for longer distances, on depending essentially on high power optical sources. As free space optical communications is a relevant research subject at INAOE, developing long distance links is of main concern. In this perspective, testing links over distances of tenths or even hundreds of kilometers normally require of beam pointing and tracking systems. Work is in progress for designing an experimental pointing and tracking system.

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